PITCH SHIFTS IN WIND INSTRUMENTS DUE TO CHANGES IN AIR COMPOSITION

Saranya Balasubramanian, Wilfried Kausel

Institute of Musical Acoustics University of Music and Performing Arts Vienna, Austria balasubramanian@mdw.ac.at

ABSTRACT

Our motivation is to understand the pitch deviation in a wind instrument that is caused by the introduction of orally expired air into a wind instrument. The effect of air composition, temperature, pressure and humidity on the speed of sound in air has been well studied for meterological purposes, and in this paper, we apply this robust model for calculating the speed of sound accounting for interaction between the gases involved and use it to calculate the acoustic impedance peaks and observe the shifts in the fundamental frequencies. In these simulations, the air composition and physical properties of air are chosen such that it closely mirrors playing conditions; and the pitch shifts are compared against experimental observations found in literature.

1. INTRODUCTION

Lip driven brass instruments or woodwinds that use reed or airjet driven instruments like flute, recorder etc. all produce sound by means of oscillating air columns inside the instrument, regardless of the excitation mechanisms. As one just begins to play these instruments, the instrument response is different to a varying degree depending on the instrument. So as a preparation for a performance, musicians typically "warm up" the instruments [1]. Some even advise exhaling before playing [2] which shows the emphasis musicians place in the effect of the air they blow into the instrument on the sound produced.

This is quite logical because the pitch is the musical counterpart of the resonance frequencies of the air column, and the resonance frequencies of air are directly determined by the wavelength and the speed of sound in air in the instrument. And it is quite well-known to musicians, who in fact play different notes by altering the length of the air-column (using toneholes, sliding valves etc. depending on the instrument) thus altering the wavelength of the sound. The speed of sound however is not intentionally controlled by the player; it is simply a property of the medium through which sound propagates. The medium in this case, air, is a mixture of ambient air and the orally expired breath of the player or completely the expired breath of the player, depending on the playing technique, instrument length and shape and the interval between playing notes. The temperature and gaseous composition of expired breath are quite different from the atmospheric composition of air, and consequently alter the speed of sound; we will see in the following sections how these factors affect the speed of sound and hence the pitch.

We will consider the gases in the mixture as real and account for interaction between the gases and their effect on the specific heat capacity of the mixture and derive the speed of sound using the methods described in [3–7]. It is important to note that in this paper, we are assuming that we know the spatial distribution of the air composition, humidity and temperature along the instrument; and attempt to quantify the effect of various physical properties of the air in the instrument to the pitch. We will then compare the results obtained with measured observations of carbon dioxide content at the mouth of the instrumentalist and the calculated pitch deviation [8, 9].

2. EXHALED AIR AND AMBIENT AIR

Parameter	Ambient Air	Exhaled Air
Temperature	21 °C	31 °C
Oxygen	20 %	15.3%
Carbon dioxide	0.0395 %	2.5% - 6%
Water vapour	0.5%	1% - 6%
Nitrogen	78%	75%

Table 1: Comparison of Physical Composition and Properties of Ambient and Exhaled Air [10–14]

2.1. Breathing

The main function of the respiratory system is to absorb oxygen and expire carbon dioxide, and is typically done through the nose. Oral respiration bypasses the filtering that happens in the nasal passage however with respect to exhaled air composition, both these processes are nearly identical. While the physiological process of breathing is the same for all individuals, the gaseous composition of ones breath also varies from one person to another, for example - heavy smokers tend to have a higher carbon monoxide content in their breath [15], varying amounts of ethanol content depending on alcohol consumption and the individual's health and fitness[11]. Table 1 shows some commonly observed differences between exhaled air and ambient air.

2.2. Air Composition

As we noted before, the main gas exchange in respiration is between oxygen and carbon dioxide. The amount of CO_2 in one breath of exhaled air is around 4% [11]; and there are reported values between 2.5% and 6% [12]. The atmospheric level of CO_2 has been steadily rising, but it is still only at 0.0395% [13]. Carbon dioxide has a molecular mass of 44.00995 grams, and thus would become the heaviest constituent in the exhaled air mixture.

The air from lungs when exhaled, passes through moist surfaces and thus has a relative humidity of nearly a 100%[16, 17]. What this means is, apart from the oxygen absorbed and carbon dioxide released from the lungs, any gap in the air composition is filled with water-vapour. The actual water vapour content value of exhaled air depends on the humidity of inhaled air [10, pp. 941-942].

2.3. Temperature

Orally expired air is less sensitive to meteorological parameters such as ambient temperature and humidity [18], however exhaled air is usually warmer than the air initially present inside the instrument owing to human body temperature.

As the air travels inside the instrument it encounters various thermoviscous losses and consequently the temperature drops. This drop across the instrument has been found to be high as $12^{\circ}C$ [19]. As one would expect, the extent of this effect depends on the instrument itself. For example, in a cornetto using the mean temperature of the air column as opposed to the temperature gradient along the bore doesn't appreciably alter the pitch [20] and on the other hand, in an experimental study of clarinets [21] a pitch difference of up to 8.5 cents was observed when the temperature gradient was replaced with an averaged temperature.

The heat diffusion is a much slower process than audio acoustic vibrations, hence the air compression and expansion resulting from oscillations behave less like an isothermal process and more like an isentropic process [22], i.e. the temperature changes will remain local, and hence it would be safer to assume a varying temperature gradient as opposed to an averaged temperature within the instrument.

3. SPEED OF SOUND

An ideal monatomic gas has only three degrees of freedom corresponding to translational motion in three dimensional space. Diatomic gases also have two rotational degrees of freedom. In ideal gases, we assume that the collisions between these molecules are completely elastic; and in atmospheric air, since Nitrogen and Oxygen are both diatomic and make up for more than 99% of its composition; it is often acceptable to ignore the interaction between these gases and treat air as an ideal diatomic gas, but in the case of exhaled air composition this would lead to inaccuracies.

Table 2 shows predicted values of speed of sound in air calculated using an approximate formula exists for speed of sound in air [23, pp. 17–18] with small amounts of additional gas, and another set of predictions using a more rigorous method (adapted from [4], described below) and the known reference values [6] for particular temperatures, and carbon dioxide and water vapour concentrations in air. The second method is decribed in greater detail below, which is more accurate will be used for all the simulations in this paper.

The ideal gas state equation, as one would expect applies only to an ideal gas, which is a theoretical construct. Real gases exhibit a more complex behaviour, owing to difference in the molecular properties of the constituent gases; and thus instead of the ideal gas state equation we will use the virial expansion upto the second virial coefficient (B). The virial coefficient is a temperature dependent quantity that accounts for the interaction between particles.

Speed of sound (c) in a mixture of real gases as derived in [4] is,

$$c = \sqrt{\gamma \, \frac{RT}{M} \left(1 + \frac{2pB}{RT}\right)},\tag{1}$$

where, γ (specific heat ratio for real gases) is,

$$\gamma = \frac{C_{\rm P}^1}{C_{\rm V}^1}.\tag{2}$$

 $C_{\rm P}^1$ and $C_{\rm V}^1$ (specific heats of real gases at constant pressure and constant volume respectively in kJ kg⁻¹ K⁻¹) are defined [4]

as,

$$C_{\rm P}^{1} = C_{\rm P}^{0} - \frac{R}{10^{3}M} \frac{p}{RT} \left(T^{2} \frac{{\rm d}^{2}B}{{\rm d}T^{2}} \right),$$
 (3a)

$$C_{\rm V}^1 = C_{\rm P}^0 - \frac{R}{10^3 M} \left(1 + \frac{2p}{RT} \left(\frac{T \, \mathrm{d}B}{\mathrm{d}T} \right) \right), \qquad (3b)$$

where,

B second virial coefficient $(m^3 mol^{-1})$

- M molar mass (kg mol⁻¹)
- R universal gas constant (J $K^{-1} mol^{-1}$)
- T thermodynamic temperature (K)

p pressure (N m^{-2})

The specific heats at constant pressure $(C_{\rm P}^0)$ for ideal gases is calculated as the weighted sum of individual specific heats $C_{\rm Pi}$, as shown below.

$$C_{\rm P}^0 = \sum_{i=1}^n C_{{\rm P}i}^0 x_i.$$
 (4)

The individual specific heats of ideal gases are approximated as a polynomial function of temperature [24]

$$\bar{C_{Pi}} = a_i + b_i T + c_i T^2 + d_i T^3$$
(5)

where,

$$\vec{C_{Pi}}$$
 ideal gas specific heat (kJ kmol⁻¹ K⁻¹)
 a_i, b_i, c_i available for common gases in [24]
T thermodynamic temperature (Kelvin)

The second virial coefficient (B in $\text{cm}^3 \text{ mol}^{-1}$) for the constituent gases is computed using the square-well function [6]

$$B_i(T) = a_i - b_i e^{\frac{c_i}{T}},\tag{6}$$

where the values for a, b, and c for the gases over applicable temperature range can be found here [25].

For a mixture of gases (excluding water vapour), we will use the following formula [4] to calculate the second virial coefficient :

$$B_{\rm aa} = \sum_{i=1}^{n} B_i x_i \frac{M_i}{M_{\rm air}}.$$
(7)

When humidity is present, the interaction virial coefficient between air and water vapour is too large to ignore, so we will use the mixing rule [6, vol. 3; p. 16] as follows

$$B = B_{aa}(1 - x_h) + 2B_{ah}x_a x_h + B_{hh}x_h^2.$$
 (8)

 $B_{\rm hh}$ can be calculated either from equation (6) or using the following formula given in [5]

$$B_{\rm hh}(T) = 33.97 - \frac{55306}{T} 10^{\frac{720000}{T^2}}.$$
 (9)

 $B_{\rm ah}$ is the interaction coefficient between dry air and water vapor, given by the following formula [5]

$$-B_{\rm ah} = 36.98928 - 0.331705T_c + 0.13903510^{-2}T_c^2 - 0.57415410^{-5}T_c^3 + 0.32651310^{-7}T_c^4 - 0.14280510^{-9}T_c^5,$$
(10)

where T_c is the temperature in Celsius. The formula holds good for $-50^{\circ}C < T_c < 90^{\circ}C$.

Figures 1 and 2 show the trend in the speed of sound against water vapour and carbon dioxide content respectively, where all the other gases are kept at to standard atmospheric composition and proportionally decreased.

			Speed of Sound (m/s)			
Temperature (°C)	$\% \operatorname{CO}_2$	$\% H_2O$	Method 1 [23]	Method 2 [4]	Actual [6]	
0	0	0	331.43	331.596	331.43	
0	0	1.21	332.032	332.065	332.06	
0	0.0314	0	331.399	331.403	331.4	
10	0	0	337.5	337.468	337.43	
10	0	2.43	338.708	338.732	338.73	
10	0.0314	0	337.469	337.433	337.4	
20	0	0	343.57	343.397	343.31	
20	0	4.62	344.917	345.867	345.84	
20	0.0314	0	343.539	343.361	343.3	
30	0	0	349.64	349.119	349.09	
30	0	8.39	352.066	353.811	353.80	
30	0.0314	0	349.649	349.082	349	
51	100	0	262.959	279.151	280	

Table 2: Comparison of predicted speed of sound with experimentally observed reference values. Humidity when not zero is the percentage of water molecules at a relative humidity of 100% at the given temperature.



Figure 1: Predicted speed of sound for varying water vapour content at different temperatures



Figure 2: Predicted speed of sound for varying carbon dioxide up to 6% (exhaled air range) at different temperatures

4. PITCH SHIFTS

Pitch is a psychoacoustic property that is perceived by the brain, and cannot be explained in isolation by the fundamental frequency alone [26]. However, since we are only interested in the deviation from the initial pitch of the cold instrument, we will restrict ourselves to fundamental frequency to mean pitch. The deviation in cents is given by the difference between the frequencies of the first impedance peak. All the simulations have been done for a very simplified wind instrument model - a cylindrical tube 100 cm long and 1 cm in diameter.

Figures 3a, 3b show the simulated pitch deviations for varying carbon dioxide, water vapour content respectively. The initial conditions in both are were identical; standard dry and carbon dioxide free air at 21 °C. The expired air will be saturated with water vapour (100% relative humidity) and the corresponding specific humidity level is between 2.5% and 6% depending on other factors such as pressure and temperature. In this case, we can see that the effect on humidity alone is very slight - less than 0.5 cents increase for a 6% concentration of water vapour in the air mixture. Carbon dioxide on the other hand shows a much stronger effect, causing the pitch to drop by over 25 cents.

Figure 3c shows the pitch shift caused due to change in temperature. The initial reference pitch is calculated for standard dry and carbon dioxide free air at 0 $^{\circ}$ C and we can see that the pitch increases linearly up to nearly 30 cents for a temperature rise of 31 degrees.

When an instrument is being played, all these effects will act together and experimentally observed [9] pitch shifts while monitoring the CO_2 and O_2 and temperature levels cannot be inferred easily by studying the trends individually. The measurements from [9] are reproduced in figure 4.

Key observations from these measurements are,

- 1. right after the attack the pitch decreases and forms a plateau
- 2. when there's a new breath, the pitch increases to its former peak value and follows the same pattern as above
- 3. when circular breathing is performed (breathing without interruption of the sound), the pitch increases suddenly as observed in the above cases
- 4. when there is a break in sound, but no renewed breath, the pitch starts from its previous value (no sudden increase) and then follows the same pattern as above



(a) Pitch shifts due to $C0_2$ content (b) Pitch shifts due to humidity in the air (c) Pitch shifts due to change in temperature

Figure 3: Simulated showing pitch shifts for varying temperature, carbon dioxide and water vapour content





(b) Measured $C0_2$ and 0_2 contents (upper part of graph) and computed expected pitch shift (lower part)

Figure 4: Relationship between carbon dioxide content and pitch shifts [9]





Figure 5: Pitch shifts simulated in a simplified instrument model - a cylindrical tube of length 100 cm, and diameter 1 cm

To simulate playing conditions, the reference pitch is calculated for the above-mentioned instrument, entirely filled with standard air at 21 °C. And then, we assume that the air inside the instrument is replaced with exhaled air as shown in figure 5a. In reality, the gases would diffuse in a complex manner and exhaled air and ambient air would not have a clear boundary as shown in figure 5a, nonetheless it is a useful means to visualise the pitch variation in terms of proportion of exhaled air content inside the instrument. In figure 5b the pitch decreases up to 10 cents when the exhaled air spreads up to nearly half the instrument. After that, the pitch starts to increase, nearly reaching the original pitch when exhaled air concentration is around 80% of the instrument, and increases still more by 10 cents when the instrument is completely filled with exhaled air. While we ignore the temperature losses along the instrument, the trend is quite similar to that observed in figure 4.

In [8], the carbon dioxide content was only measured in the player's mouth and gives no information about the spatial propagation of the gases; but considering the fact that the volume of air in a single breath cannot fill the instrument, and that the diffusion is not instantaneous, it seems dubious to assume that the gas distribution along the entire instrument would be identical to that measured in the player's mouth for calculating the resonant frequencies. Figure 5b does not show an increase in pitch at every attack though, the pitch is observed to increase only when the instrument is at least half filled with exhaled gas; which maybe the case from the second breath in the measurements in figure 4.

5. CONCLUSIONS

By establishing the spatial distribution of gases and their physical properties as an input parameter in calculating the pitch, we are excluding the modelling of propagation and diffusion of gases (which is a complex subject in its own right, theoretically modelled in [27]), localised temperature changes due to compression and rarefaction of the air during oscillations, thermoviscous losses near the wall and lossy boundaries from the scope of this exercise. It appears that along with varying air composition, the spatial distribution of gases too play a part in the pitch shifts observed; and needless to say, this needs to be verified experimentally.

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