ANALYTICAL MODEL OF THE TRANSITION BETWEEN CYLINDER AND CONICAL TUBES

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ABSTRACT

For musical wind instruments, the problem of the transition between a cylindrical tube and a conical one occurs in many situations. In a sufficiently long cylinder, only planar waves can propagate, while in a sufficiently long cone, only spherical waves can propagate. Modelling the matching between the two tubes requires the consideration of higher-order modes, which are evanescent, i.e., close to the matching volume. A mode-matching method yields complicated computation, thus a numerical discretization method (such as the Finite Element Method, FEM), or experiment are more suitable. However the general shape of the model is known, under the condition that only one mode propagates in each tube, i.e., that the frequency is low enough. The model includes the effect of compressibility of the matching volume, and three acoustic masses. The major result is that the latter are linked together by the geometrical parameters, therefore only one mass has to be determined numerically or experimentally. It is proportional to the so-called length correction. Its knowledge allows the determination of an analytical model valid up to the first cutoff frequency. The results of the analytical formulas are compared with those of the FEM method, and the agreement is very good. The literature on this subject is rare, except the papers by Chester (J. Sound Vib. 1987) and Martin (J. Eng. Math., 2011). Comparison with formulas found in these papers is carried out, and it is found that the present study improves their results. Furthermore, for the particular case of a cylindrical tube radiating with an infinite flange, the analytical formula compares very well with the exact result (Norris and Sheng, J. Sound Vib. 1989), up to the first cutoff frequency.