

PLAYING SLOW IN REVERBERANT ROOMS – EXAMINATION OF A COMMON CONCEPT BASED ON EMPIRICAL DATA

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ABSTRACT

During the performance of music the room acoustical environment has a substantial influence on the player's perception of the music. This presumably affects his way of playing, the more so if one assumes an inner reference that he wants to convey to the audience. Reducing the tempo in very reverberant rooms is a strategy that is often reported by musicians and it was, for example, recommended by J. J. Quantz in his famous music treatise in 1752. In this paper, the data collected in a field study conducted with a cellist in 7 European concert halls and a laboratory study conducted with 12 musicians in 14 virtual performance spaces is used to investigate in how far this strategy is actually followed in practice. A software-based analysis of the recordings of the musicians as well as room acoustical measurements in the halls were the basis for a statistical analysis of the influence of parameters like T_{30} , or ST_{late} on the tempo of the pieces played in each concert hall. The results suggest that the adjustment of tempo strongly depends on the basic tempo of the performed music and that there are different types of strategies adopted by musicians. These are elucidated by statements collected in interviews that were conducted with the performers during the experiments.

1. INTRODUCTION

The way musicians perceive their own tonal rendition depends on the rooms they perform in, since the acoustics of a performance space has a strong effect on the sound of the music that is played. It is likely that performers make certain adjustments in their way of playing to adapt to the room acoustical surrounding. An interrelation that is frequently named by musicians is that reverberant rooms require a reduced tempo. In his work on flute playing, already J. J. Quantz gave a detailed description of how notes are blurred if they are played too fast in reverberant rooms and advocated a reduced tempo under these conditions [1]. This kind of adjustment was similarly recommended by music scholars in their treatises of the 20th century [2] [3]. There are only few empirical investigations on the influence of reverberance on the playing tempo of musicians and they have yielded partly conflicting results. An early study carried out with the Cleveland Orchestra used the duration of whole movements as measure for the tempo played in halls with reverberation times of between 1 s and 2.1 s [4]. The results did not confirm the expected negative linear relation between tempo and reverberation time but instead implied a negative correlation between the room acoustical quality and the duration of the movements. An investigation using a MIDI grand piano played by different performers in a room with variable acoustics used the inter-onset-intervals of the recorded MIDI tones as measure for the tempo [5].

Surprisingly, no influence of any of the room acoustical parameters (reverberation time, late reverberation level, ratio between direct sound and early reflections, spectral properties of reverberation) on the mean tempo of the music pieces could be demonstrated. In a laboratory study carried out in an anechoic room, soloists of different instruments played musical phrases in room acoustical environments simulated with 6-channel loudspeaker reproduction [6] [7]. Interestingly, most of the musicians not only reduced their playing tempo (measured by the mean phrase duration) in very reverberant rooms but also under anechoic conditions.

All studies mentioned above were focused on the reverberation time as the only room acoustical measure for reverberance, disregarding other features that are perceptually more relevant from the performer's perspective. Moreover, a sufficiently large variance of room acoustical conditions should be provided in order to cover the whole range of possible performance venues and to identify also non-linear interrelations.

In the studies presented here, two different approaches were taken to further explore the relation between reverberance and tempo: A case study with a renowned solo cellist was carried out in real concert halls and a laboratory study using computer models of concert spaces that were auralised by means of dynamic binaural synthesis was conducted with twelve performers of six orchestral instruments. In both studies, the musicians' performances of different pieces were recorded, a software-based analysis of the recordings was employed to quantify the tempo of the music performances and room acoustical parameters typically used to predict reverberance were determined in the performance spaces. This was the basis of a statistical analysis designed to investigate the effect of the room acoustical conditions on the tempo, thereby taking into account the influence of the basic tempo of the performed pieces and, in the laboratory study, the played instrument and the performers' individuality. The two studies are described in detail in [8] and [9], respectively.

2. METHODS

2.1 Recordings and performance analysis

The case study was carried out in collaboration with the cellist Jean-Guihen Queyras who performed the *Six Suites for Violoncello Solo* by J. S. Bach in seven European concert halls (6 suites \times 6 movements = 36 pieces). He was recorded with a boundary layer microphone (Schoeps BLM 03 C) that was placed 11.5 cm from his cello spike, so it was positioned almost in the centre of the hemispherical directivity of the microphone. Thus, the distance between instrument and microphone remained constant and inclinations of the cello during the performances did not cause any loudness fluctuations in the recordings. Because of the short distance between instrument and microphone the influence of the sound of the rooms could be neglected.

The laboratory study took place in the fully anechoic chamber of the TU Berlin ($V = 1850 \text{ m}^3$, $f_c = 63 \text{ Hz}$) and was conducted with six instruments each played by two professional solo musicians: violin, cello, clarinet, bassoon, trumpet and trombone. The musicians played excerpts of approx. 1 minute duration of two pieces of their own choice that had a calm and lively character, respectively (see [9] for a list of the pieces). They were recorded with a miniature microphone (Sennheiser MKE 1) that was attached directly to the instruments (see Figure 3).

The method for deriving a description of the tempo chosen by the musicians in their performances on the basis of the recordings was aimed at a perceptually meaningful analysis. By means of a software-based analysis five technical tempo related features on note- and beat-level were extracted from the software [10]. To predict the tempo of musical pieces as perceived by listeners, two of these technical features were used as predictor variables in a regression formula with 90% explained variance that was obtained in [11].

2.2 Room acoustical parameters

In all the studies on the effect of room acoustics on music performance mentioned above, the reverberation time RT was used as a measure for the duration of reverberation. When evaluating the comments and recommendations of musicians and music scholars, it is however not entirely clear, if it is the total duration of reverberation that mostly influences the choice of tempo. According to [12] the early decay time EDT and the late support ST_{late} are more appropriate parameters to predict the reverberance perceived by listeners and musicians, respectively. The late sound strength G_1 was recently recommended as a parameter similar to ST_{late} but with less measurement uncertainty [13]. Finally, the blurring of tones as described by Quantz might also be related to the clarity of a concert hall, which is predicted by the parameter C_{80} . To investigate the influence of reverberance on the musical tempo, all of the five above mentioned room acoustical parameters were taken into account.

In the field study, room acoustical measurements according to [12] were carried out in the seven concert spaces (see [8] for a detailed description of the halls). The aim of the study was to determine the room acoustical conditions during the cellist's concerts as exactly as possible in order to draw valid conclusions about their influence on his way of playing. Thus, computer models of the concert spaces were constructed and their room acoustical properties were fitted to the measurements taken in the real halls (this routine is described in detail in [8]). Then, the presence of an audience as well as a source with the directivity of a cello was simulated in the models [14] [15] before carrying out room acoustical measurements. The directional source was thereby placed at 0.6 m height at the same stage position where the cello had been during the concerts and the receiver was positioned at 1.2 m height and 0.4 m from the source. The five room acoustical parameters used as independent variables in this paper were determined with the above source-receiver-setup and are denoted by the subscript vlc in the following. The values measured for the concert spaces of the field study are shown in Figure 1.

The 14 concert halls simulated in the laboratory study were computer models based on existing halls [16] [17] [18] [8] representing typical performances spaces for Western classical music (see [9] for a detailed description of the halls). The room acoustical properties of these models were determined with a receiver placed at the centre of each stage 2.5 m from the stage edge at a height of 1.2 m and with

sources conforming to the directivities of the instruments involved in the experiments [14] [15] that were placed at their respective typical distance from the receiver [9]. The parameters measured in the room models using the described source-receiver-setup are denoted by the subscript ins in the following. The five parameters used in this paper to describe the reverberance of the simulated halls are shown in Figure 2.

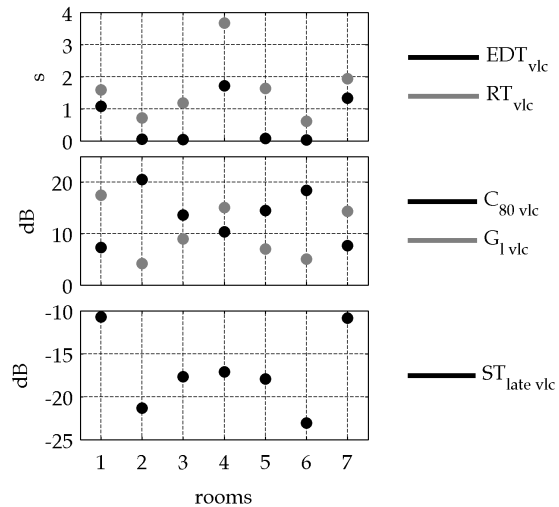


Figure 1: Room acoustical parameters measured on stage of the concert halls of the field study using a source with the directivity of a cello and a source-receiver distance typical for a cellist and his instrument.

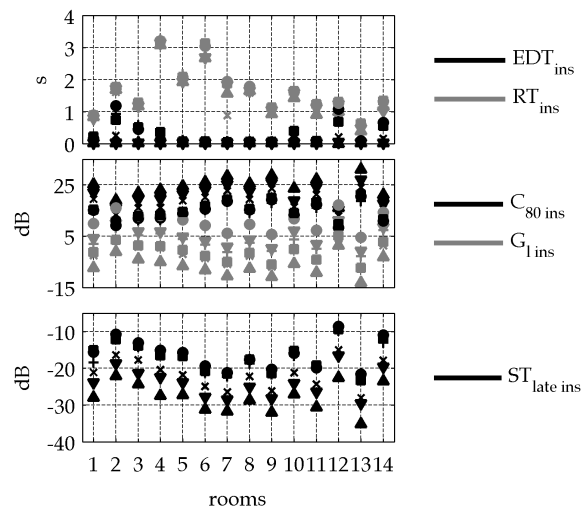


Figure 2: Room acoustical parameters measured on stage of the concert halls of the laboratory study using sources with the directivity of the involved instruments (\times : Vl, $+$: Vlc, ∇ : Cl, \blacktriangle : Trp, \bullet : Trp, \blacksquare : Trb) and a source-receiver distance typical for a musician and his instrument.

2.3 Experimental procedures

The cellist in the field study was recorded during his regular performances of the *Six Suites for Violoncello Solo*, which took place over the course of one year. After each concert, a guided interview was conducted, in which he was asked to describe the room acoustics of the hall he had performed in as well as his way of playing and conscious adjustments that he had made.

The room acoustical environments in the laboratory study were simulated on the basis of the computer models by means of dynamic binaural synthesis [19]. The method for acquiring binaural room impulse responses (BRIRs) from the models is

described in detail in [9] and yielded one dataset for each of the 14 rooms for head rotations between $\pm 50^\circ$ and head elevations between -30° and 21° with a resolution of 2° and 3° , respectively. During the experiment, the head movements of the musicians were detected by head tracking (Polhemus Patriot) in order to select the appropriate BRIR from the dataset and to convolve it in real-time with the anechoic input signal recorded at the instrument. It was only the response of the room that was thereby simulated, since the direct sound of the instrument was produced in the real experimental environment. The simulation was presented to the performers with extra-aural headphones (AKG K-1000) that barely impeded the instrument's direct sound path to the performer's ears. The frequency responses of the recording microphone and the headphones were equalised, the latter was compensated individually for each musician [20]. Prior to the recording session, the performers were given 10 minutes to become familiar with each virtual room. Then, they were recorded playing excerpts of two music pieces and interviewed with the same questions that had been used in the field study. The warm-up, the recording of the same two pieces and the guided interview was repeated in each of the randomly presented virtual rooms.



Figure 3: Left: Microphone attached to a violin in the laboratory study. Right: Violinist playing in a simulated concert hall while wearing extra-aural headphones.

2.4 Statistical analysis

The aim of the statistical analyses was to reveal the effect of the five room acoustical parameters on the tempo chosen by the musicians when playing the 36 pieces in 7 concert halls (field study) and the 2 pieces in 14 concert halls (laboratory study) in separate analyses for each of the two datasets. By using hierarchical linear models (HLMs) as analysis method [21], the nested structure of the data (pieces, rooms / pieces, musicians, rooms) was accounted for, since variances were considered on each level of the data.

Before the actual analysis, the variances on each level were compared separately for both datasets by using intercept-only HLMs, i.e. models with no regressors that only consider the grouping structure of the data. This showed that for the laboratory dataset the variance on the room level was very small compared to the variance on the other levels. This implies that the variance of the musicians' individual adjustments to the room acoustics was greater than the variance of their averaged adjustments, i.e. that the players' reaction patterns to the room acoustical environment were highly individual. Since the room level variance was so small, indicating that the room level was not relevant in the hierarchical structure, this level was omitted in the further analysis [21].

Since high correlations between the five room acoustical parameters would have caused problems of multicollinearity, the parameters were not used as five predictors in single HLMs but rather as single predictors in five HLMs. Furthermore, the results of previous studies have indicated that the relation between tempo and reverberance might be

inversely quadratic [6]. To explore this evidence, the room acoustical parameters were used as squared predictors in further HLMs. In total, ten HLMs were calculated for each dataset with the restricted maximum likelihood method and standardised independent and dependent variables by using one of the five room acoustical parameters shown in Figure 1 and Figure 2 either as squared or as linear predictor.

The resulting models were compared by calculating their explained variance on the room and musician level, respectively, with the following formula [22]:

$$R^2_{\text{level 2}} = 1 - \frac{\sigma_{M1|\text{level 2}}^2 + \frac{\sigma_{M1|\text{rest}}^2}{n}}{\sigma_{M0|\text{level 2}}^2 + \frac{\sigma_{M0|\text{rest}}^2}{n}} \quad (1)$$

$\sigma_{M1|\text{level 2}}^2$ and $\sigma_{M1|\text{rest}}^2$ are the level 2 (room / musician) variance and the residual variance of the target model M1. $\sigma_{M0|\text{level 2}}^2$ and $\sigma_{M0|\text{rest}}^2$ are the variance on the respective levels in an intercept-only model M0 with no predictors. n is the number of groups (rooms / musicians).

3. RESULTS

Figure 4 and Figure 5 show the regression coefficients β (black *) with 95% confidence intervals for the single regressors in each HLM calculated with the data of the field and the laboratory study, respectively. The figures show the regression coefficients for that version of each predictor (linear or squared) that had the higher explanatory power. Each β illustrates the extent and significance of the effect of each room acoustical predictor on the tempo derived from the recordings with CIs not crossing the zero-line being considered as a significant effect.

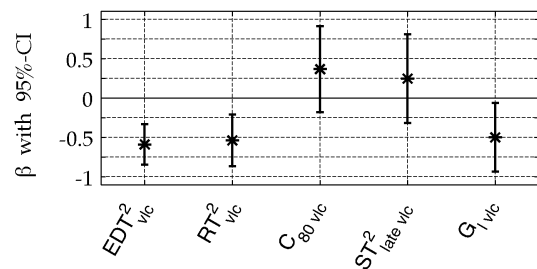


Figure 4: Field study – Standardised regression coefficients with 95% confidence intervals (CIs) for the five room acoustical parameters (x-axes) predicting the response variable tempo. CIs not crossing the zero-line indicate significant coefficients ($p < 0.05$).

As can be seen in Figure 4, the cellist in the field study significantly reduced his tempo in rooms with long EDT_{vlc} and RT_{vlc} as well as high G_{vlc} . Similar to the results of [6], he also reduced his tempo with low EDT_{vlc} and RT_{vlc} . The difference between the effects of EDT_{vlc} and RT_{vlc} on the one hand, describing the duration of reverberation (quadratic relation) and G_{vlc} on the other hand, describing the strength of reverberation (linear relation), indicate that these parameters are related to different perceptual qualities on the side of the musician. It is noteworthy that the strongest effect on tempo can be observed for EDT_{vlc}^2 with an explained variance of 74.36%. In the interviews conducted after the concerts, the cellist not only reported to slow down in reverberant environments to avoid a muddled sound, which is in line with Quantz. He furthermore mentioned the prolongation of pauses under such conditions, which additionally contributes to a reduced overall tempo. His

slowing down in rooms with short duration of reverberation might be due to the prolongation of notes as a strategy to compensate for the lack of sound decay, as it was addressed by other musicians in [23].

Figure 5 shows that in the laboratory study the *tempo* averaged over musicians and pieces was significantly reduced in rooms with long RT_{ins} and significantly increased in rooms with high and low G_{1ins} . Again this indicates a separate perception of duration and strength of reverberation, but the linear and quadratic relations between these aspects and *tempo* are contrary to the results of the field study. Furthermore, a strong tendency for a positive effect of C_{80ins}^2 and $ST_{lateins}^2$ can be observed. The strongest effect on the mean *tempo* of the musicians of the field study can be observed for RT_{ins} . It is apparent that the effects in Figure 5 are much smaller than in Figure 4 (different scaling of the y-axes). This is due to the fact that the HLM coefficients in Figure 5 were calculated for the *tempo* averaged over musicians and pieces, while at the same time the adjustment strategies of the performers were very individual. To be able to compare the explanatory power of the predictor RT_{ins} to the one of EDT_{vic}^2 as predictor for the data of the field study, an HLM was calculated for the laboratory dataset with RT_{ins} as predictor and the individual musicians as additional factor to take into account their individual adjustment strategies. This resulted in an explained variance of 8.73%.

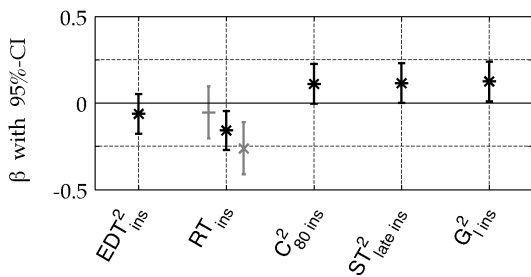


Figure 5: Laboratory study – Standardised regression coefficients with 95% confidence intervals (CIs) for the five room acoustical parameters (x-axes) predicting the response variable *tempo*. The coefficients of the HLM with room acoustical predictors only are shown as black *, coefficients of an HLM with ‘basic tempo’ as additional factor are shown in grey (–: ‘fast’, ×: ‘slow’). CIs not crossing the zero-line indicate significant coefficients ($p < 0.05$).

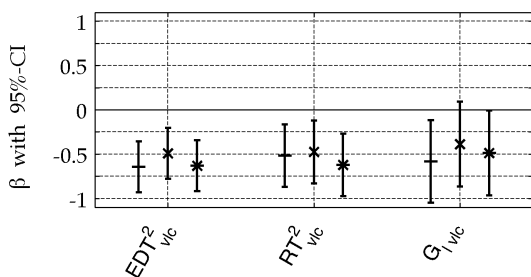


Figure 6: Field study – Standardised regression coefficients with 95% CIs for the room acoustical parameters separately predicting the *tempo* of the movements classified by the factor ‘basic tempo’. –: variable, ×: slow, *: fast. CIs not crossing the zero-line indicate significant coefficients ($p < 0.05$).

To investigate the interaction of the basic tempo of the played pieces with the influence of the room acoustical

parameters on *tempo*, the movements of each suite played by the cellist in the field study were classified as ‘variable’ (movements 1 and 5), ‘slow’ (movements 2 and 4) and ‘fast’ (movements 3 and 6). Then, 3 HLMs were calculated by using each of the significant room acoustical parameters shown in Figure 4 as predictor and the basic tempo as additional factor. Figure 6 shows the resulting regression coefficients with 95% CIs and illustrates that the effect of the reverberance on *tempo* was slightly stronger for the movements with ‘variable’ and ‘fast’ basic tempo, while the ‘slow’ movements were less affected.

The grey – and × in Figure 5 show the regression coefficients of an HLM with RT_{ins} as predictor and the basic tempo of the two pieces (‘fast’, ‘slow’) played by the performers in the laboratory study as additional factor. The difference between the two pieces regarding the effect of the other room acoustical parameters was very small, so these coefficients are not shown here. It becomes apparent that on average the musicians played the slow pieces considerably slower than the fast pieces in rooms with long RT_{ins} . In the guided interviews of the laboratory study, almost all musicians reported to prolong pauses in reverberant environments. However, it was explained that this strategy was used primarily for slow pieces, while for fast pieces the focus rather lay on clear, short articulation. Thus, these statements support the statistical evidence very clearly.

In Figure 7 the influence of RT_{ins} on the ‘slow’ and ‘fast’ pieces played in the laboratory study is shown separately for each instrument. It can be seen that it was the cello, the bassoons, the trumpets and the trombones who significantly reduced the *tempo* of the slow pieces in rooms with long reverberation time. The *tempo* of the fast pieces played by the bassoons and the brass players was not affected by RT_{ins} . In the case of the cello, the short articulation described above as a strategy for fast pieces in reverberant environments was apparently accompanied by a faster *tempo*, which was confirmed by an interview statement of one of the cellists in the study. The clarinets did not adjust the *tempo* of any of the pieces to the reverberation time of the rooms while the violins reacted similarly to the cellist of the field study: They played the fast pieces slower with increasing RT_{ins} , while the *tempo* of the slow pieces was not adjusted.

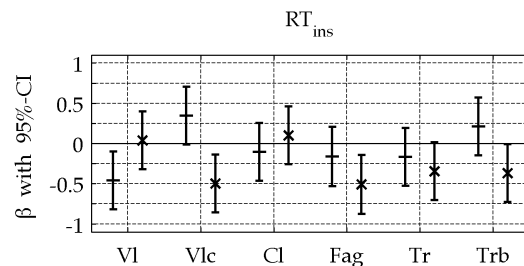


Figure 7: Laboratory study – Standardised regression coefficients with 95% CIs for RT_{ins} separately predicting the *tempo* of each instrument (x-axes) and each piece (–: ‘fast’, ×: ‘slow’). CIs not crossing the zero-line indicate significant coefficients ($p < 0.05$).

4. CONCLUSIONS

On the basis of data collected in a field and in a laboratory study, statistical analyses with hierarchical linear models were carried out in this paper to elucidate the effect of five room acoustical parameters describing different aspects of reverberance on the *tempo* played in music performances.

Based on their effect on the way of playing, the duration and the strength of reverberation were obviously perceived as different acoustical qualities by the musicians. Significant effects were

found for the early decay time, the reverberation time and the late sound strength, while the clarity and late support seem to be indicators of minor importance. For the data of the field study, the strongest influence was shown for EDT_{vlc} ($R^2 = 74.36\%$), for the data of the laboratory study it was RT_{ins} ($R^2 = 8.73\%$). The amount of variance explained by a single room acoustical parameter in the field study is quite surprising since in real concert situations there are many other factors potentially influencing the performance. It must be considered, though, that some of these factors might be correlated with the room acoustics, for example, visual impressions of the size of the room and the stage. Thus, the auditory and visual impressions taken together might have a stronger impact than the isolated auditory information alone. The latter was the case in the laboratory study, where the musicians additionally had to allocate cognitive capacity in order to imagine the simulated room they heard and to get used to the experimental situation. This might explain the considerably lower amount of explained variance reported above.

The results of the field study data showed a significant negative effect of EDT_{vlc} , RT_{vlc} and $G_{1\text{vlc}}$, while the effect of these parameters was slightly lower for the slow movements than for the movements with fast and variable basic tempo. Averaged over musicians, a significant negative effect of RT_{ins} and a significant positive effect of $G_{1\text{ins}}$ on the tempo were found for the laboratory study data. On average, the musicians of the laboratory study played the slow pieces with reduced tempo with increasing RT_{ins} , while the tempo of the fast pieces was not adjusted. Comparing the effect of RT_{ins} on the two pieces for the six instruments showed clear differences, implying specific adjustment strategies of the players.

In summary, this study was able to confirm that the reverberance of performance venues has significant effects on the playing tempo of musicians. The strategies how to adapt to the acoustical environment, however, seem to be largely individual. While some musicians reduce the tempo only for faster pieces to avoid a muddled sound, others seem to put more focus on a short articulation with an even faster tempo as a possible consequence. Moreover, the interrelation between reverberance and speed is not necessarily linear: Some musicians (as the cellist in the field study) use a slower tempo also in very dry room acoustical environments, presumably because of more prolonged tones as a compensation for a lack of room acoustical decay.

5. ACKNOWLEDGEMENTS

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6. REFERENCES

- [1] J. J. Quantz, Versuch einer Anweisung, die Flöte traversière zu spielen, Kassel et al.: Bärenreiter, 1983.
- [2] I. Galamian, Principles of Violin Playing and Teaching, Englewood Cliffs: Prentice-Hall, 1962.
- [3] P. Borciani, Das Streichquartett, Milan: Ricordi, 1973.
- [4] F. F. Winckel, "Optimum Acoustic Criteria of Concert Halls for the Performance of Classical Music," *J. Acoust. Soc. Amer.*, vol. 34, no. 1, pp. 81-86, 1962.
- [5] S. Bolzinger, O. Warusfel and E. Kahle, "A study of the influence of room acoustics on piano performance," *Journal de Physique IV, Colloque C5, supplement au Journal de Physique III*, vol. 4, pp. 617-620, 1994.
- [6] K. Ueno, K. Kato and K. Kawai, "Effect of room acoustics on musicians' performance. Part I: Experimental investigation with a conceptual model," *Acta Acustica/Acustica*, vol. 96, pp. 505-515, 2010.
- [7] K. Kato, K. Ueno and K. Kawai, "Effect of room acoustics on musicians' performance. Part II: Audio analysis of the variations in performed sound signals," *Acta Acustica/Acustica*, vol. 101, no. 4, pp. 743-759, 2015.
- [8] Z. Schärer Kalkandjiev and S. Weinzierl, "The influence of room acoustics on solo music performance: An empirical case study," *Acta Acustica/Acustica*, vol. 99, no. 3, pp. 433-441, 2013.
- [9] Z. Schärer Kalkandjiev and S. Weinzierl, "The influence of room acoustics on solo music performance. An experimental study," *Psycholmusicology: Music, Mind and Brain*, 2015.
- [10] A. Lerch, Software-Based Extraction of Objective Parameters from Music Performances, Munich: Grin Verlag, 2009.
- [11] S. Weinzierl and H.-J. Maempel, "Zur Erklärbarkeit der Qualitäten musikalischer Interpretationen durch akustische Signalmaße," in *Gemessene Interpretation. Computergestützte Aufführungsanalyse im Kreuzverhör der Disziplinen*, H. von Loesch and S. Weinzierl, Eds., Mainz et al., Schott, 2011, pp. 213-236.
- [12] ISO 3382-1, Acoustics - Measurement of room acoustic parameters - Part 1: Performance spaces, Geneva: International Organization for Standardization, 2009.
- [13] J. J. Dammerud, Stage Acoustics for Symphony Orchestras in Concert Halls, Doctoral Thesis, University of Bath, 2009.
- [14] R. Schneider, Simulation einer musikalischen Aufführung durch die Auralisation von Musikinstrumenten mit korrekter Richtcharakteristik, Master's Thesis, Technical University of Berlin, 2011.
- [15] M. Pollow, G. Behler and F. Schultz, "Musical instrument recording for building a directivity database," in *Fortschritte der Akustik - Tagungsband der 36. DAGA*, Berlin, 2010.
- [16] S. Weinzierl, Beethovens Konzerträume. Raumakustik und symphonische Aufführungspraxis an der Schwelle zum modernen Konzertwesen, Frankfurt am Main: Verlag Erwin Bochinsky, 2002.
- [17] L. Beranek, Concert Halls and Opera Houses - Music, Acoustics and Architecture, 2nd ed., Berlin et al.: Springer, 2004.
- [18] T. Hidaka and N. Nishihara, "Objective evaluation of chamber-music halls in Europe and Japan," *J. Acoust. Soc. Amer.*, vol. 116, no. 1, pp. 357-372, 2004.
- [19] A. Lindau, T. Hohn and S. Weinzierl, "Binaural resynthesis for comparative studies of acoustical environments," in *Proc. of the 122nd AES Convention*, Vienna, 2007.
- [20] A. Lindau and F. Brinkmann, "Perceptual evaluation of headphone compensation in binaural synthesis based on non-individual recordings," *J. Audio Eng. Soc.*, vol. 60, no. 1/2, pp. 54-62, 2012.
- [21] J. Hox, Multilevel Analysis, 2nd ed., New York/Hove: Routledge, 2010.
- [22] T. A. B. Snijders and R. J. Bosker, "Modeled variance in two-level models," *Sociological Methods & Research*, vol. 22, no. 3, pp. 342-363, 1994.
- [23] D. Blum, The Art of Quartet Playing. The Guarneri Quartet in Conversation with David Blum, Ithaca: Cornell University Press, 1987.