MODAL ANALYSIS OF VIOLONCELLO TAILPIECES - COPIES DERIVED FROM 3 TAILPIECES ASCRIBED TO STRADIVARI (MUSEE DE LA MUSIQUE PARIS)

Anne Houssay (1) & Eric Fouilhé (2)

(1) Musée de la musique's Laboratory, Paris
(2) LMGC Montpellier 2 University, CNRS UMR 5508
ahoussay@cite-musique.fr

ABSTRACT

In previous publications, we identified the vibrating modes of a cello tailpiece free and under string tension on a dead-rig (2009-2010), using hammer and accelerometer with George Stoppani's software, and then its behaviour on a cello. We compared different measurement methods and measured the variability of wood choices on vibration modes and frequencies (2011 - 2012). The adjusted position of the tailpiece has also been explored, by varying the "after-length", i.e. the distance of the tailpiece to the bridge which leaves a small length of vibrating string (2013). We showed that, even if less mentioned by luthiers and musicians, the distance of the attachment at the bottom of the instruments has more influence on the modes and on the sound than the "afterlength". Our study took a more historical path to identify the trends and theories on the "afterlength" significance (2014). We showed that the "afterlength", often discussed as an adjusting parameter of the sound, became an issue especially when the industrialization process in Markneukirchen enhanced the standardization of tailpiece length, with the consequence of a loss of experience on the violin makers' part, which stopped altogether making the tailpieces themselves.

Different historical types of tailpieces of the 17th and 18th century assigned to Stradivari which show different stages in historical set up at a time of transformation of the lower register instruments in the violin family are studied. Made on the same outline, their modal analysis free-free as well under the string tension on a dead-rig helps us to compare their behaviour.

1. INTRODUCTION

The most important change for string musical instruments in the 17th century is a change in paradigm concerning the understanding of vibrating strings.

Figure 1: Parameters of vibrating strings until the 17th century.

The mathematics explaining the tuning of string were ascribed to Pythagorus since ancient Greece, linking tunig to the length, the tension, and the "fatness" of the string, which lead to a theory of proportions described by the monochord. The tensions were measured by weights hanged to the nut of the monochord [1]. Different materials, mostly metals (copper, silver, gold, etc.) symbolised by different planets gave different notes, associated with different lengths. Musical treatees of meadieval times, latin as well as arabic, took the same theory for the complete story [2]. These ratios were used for musical intervals also and for designing purposes, in architecture as well as for the making of musical instruments well into the Renaissance. The questioning of musical scales preoccupied greatly music theorists during the italian Renaissance, as they tried to define consonances and dissonances. [3]

The discovery came from Vincenzo Galilei, a great luthenist and theorician, and his son Galileo. They spent a part of the summer 1592 measuring strings and weighing them, and were the first to calculate the influence of the density of the material of a musical string, or more precisely the mass per unit length, or linear mass **μ**. [4] which came to the mathematical expression :

$$\mathbf{f = (1/(2L))*\sqrt{(T/\mu)}}$$

This discovery was a threat to the authority of the Catholic Church, then strongly involved with *Counter Reformation*. God's creation of a coherent and beautyfull world was explained with theories of the Celestial Spheres and of Universal Harmony, with the beauty of the ratios of whole numbers. This paradigm coud'nt safely be contradicted at the time, thus leading Galileo to well known difficulties. In consequence, this musical discovery was to be published more than fourty years later, four years before Galileo's death in 1638. The book was published in Leyden, were the main University of the Reformed Netherland Republic had its site. The text was immediately translated in French by father Mersenne, who had published his "Harmonie Universelle" only two years earlier [5].

So heavier strings could lower the sound dramatically, and instruments could be made shorter for the same tessiture. A harpsichord's tuning could be lowered by an octave with golden strings instead of normal iron and copper strings, experiment which was tempted at the Medici court [6] [7]. By 1664, wound strings were for sale at Playford's shop in London, and shortly elsewere.
In consequence, lower instruments could me made shorter. The violoncello replaced the Bass violin, and the lower contrabass appeared in Paris around 1700, as musical answers to the technical wounded strings [8]. Cellos tuned likee Bass violins had much more manageable string lengths and Bass violins were often recut to satisfy the new demand [4].

In this context, Antonio Stradivari came at the right time to redesign not only the solo violin but also the new violoncellos, as well as their fittings. Historical tailpieces are kept in the Musée de la musique's collections and give evidence to his experiments. Changes Stradivari was bringing for the second age of the baroque are technically very informed [9]. As Tony Faber states: "After a reconstruction of the design of violins between 1685 and 1709, Stradivari also started to apply the new features that were such a succes for "big" sounding violins to cellos: flatter archings, adapted length (29 inches), to meet the challenge of tone projection and a balance between trebble played in higher positions, and the bass strings. After a six-year gap in the making of cellos, he started again with new designs; These models became "a template for generations of cello –makers up to our days. "

2. DESCRIPTION OF TAILPIECES ASCRIBED TO STRADIVARI AND CHOICE OF MODELS FOR TESTS

2.1 Differences between the original tailpieces

In the Musée de la musique's collection, violin tailpieces and four larger tailpieces are ascribed to Stradivari. They have most probably be taken off instruments of the master brought to Paris after the Napoleonic Italian wars. J.-B. Vuillaume himself made trips to Italy to buy instruments, as other musicians and dealers as well. Three of these tailpieces (**E.487, E.486.1 and E.486.2)** were given to the *Musée Instrumental du Conservatoire de Paris* by violin maker and dealer Jean-Baptiste Vuillaume [11]. The inlayed maple tailpiece **E.619** was given by the violin maker and dealer Eugène Gand in 1874. [11].

Research in the iconography has showed us a probable chronological order (see some examples in our historical study [14]).

Figure 2: **E.487, C.161** [1]**:** A. Stradivari. Curved and carved walnut wood (?) for a middle size 'contralto' or 'tenor' violin. Being of 210 mm, it is quite a lot shorter than the others and will not be part of this study on cello tailpieces, but it is worth noting that Stradivari probably made this type of tailpiece also for larger basses at the beginning of his career.

Figure 3: **E.619,** C.193: Violoncello tailpiece**,** c. 1700**,** A. Stradivari. Plain maple with purfling inlays. Carved in inlayed solid maple, with original attach in gut passing through holes drilled in the top surface

Figure 4: **E.486.1** C.114. Violoncello tailpiece**,** A. Stradivari around 1710. Maple with an inlayed ebony veneer (a thin plate wood glued on the lowered surface) surrounded by a white purfling leaving the maple at the edge.

Figure 5: **E.486.2**, C.114. Violoncello tailpiece**,** A. Stradivari, 18th c**.**. In plain maple with complete thick ebony veneer in one piece all over the surface.

2.2 Types of attachment

Different attachments were used by Strradivari from simple holes, which are in use for many baroque violins of all sizes and often seen on iconography, but also attachments which are not visible from the top, where guts or metal wires are inserted in longitudinal holes.

Figure 6: E. 619 : Simple holes trhough square lump.

Figure 7: E. 486.1 : square lump with longitudinal holes for thick gut; E. 486.2 : Square lump with longitunal holes for through attachments in brass

2.3 Choices of models for the tests

In this transition period when the violoncello is changing status from a part of the violin "concert" to a continuo and soloist instrument, these changes participate to the innovations from Bass violin to violoncello. Let us study their differences in dimensions and decide the size, weights, materials and structure of our models made for the tests.

Measures in mm			**Tested models**	
	E619	**E486.1**	**Tailpieces made for analysis**	**E486.2**
Total length	235.5	238	238	243
Width mini	30	31	32.5	30.2
Width at bottom	36.6	40.3	40.3	39.9

Figure 8: main measurements.

Here, if we assume that the plane maple tailpiece is the oldest, it is nevertheless the shortest. and the three are not exactly proportional in dimensions.:

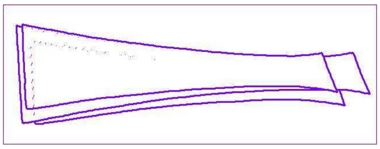

Figure 9: These three Stravivari's tailpieces : **E619, E486.1, and E486.2** are made of different lengths and proportions.

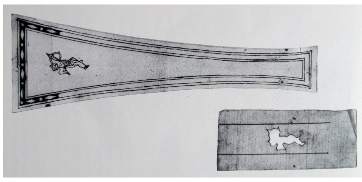

Figure 10: Design of a contralto tailpiece Stradivari 1690 (Sacconi). *Museo del Violono, Cremona* Notice that there is no attachment hole visible on the top at the smaller end. Sacconi explains that Stradivari made the tailpieces made proportionally to the instruments [9].

2.4 Weights and thicknesses

If we are right with the chronological order, there is a deliberate increase in the weight and thicknesses from one to the next. The use of first a thin ebony veneer, and then a thicker one, makes the piece even stronger, stiffer and heavier.

Strad made the tailpieces with increasing weights (the last piece contains remains of a brass wire which makes it even heavier). The first two have comparable thicknesses, the last piece is deliberatly thicker all over by more than a mm, and heavier at the bottom with the lump of attachment 3 mm thicker.

Measures in g and mm			**Tested models**	
	E619	**E486.1**	**Tailpieces made**	**E486.2**
Weight	48g	64.6g	Different weights, similar	79.9g – (without bass plate and

				wire)
Thickness center of head	11.2	10.3	10.3	12.4 at top holes 10.3 at top edge
Thickness in the middle	8.9	8.2	8.2	9.9
Thickness at the minim. width	7.7	7.8	7.8	9.2
Total max. thickness at lump	11.5	11.6	11.8	16.5

Figure 11: Main dimensions of the origninal tailpieces and choices for tested models..

We haven't done the much heavier lump at the end of the piece because of the way the copies were produced, i.e. with the same geometry. It would be worth trying another time, though.

3. MAKING THE TAILPIECES FOR TESTING

3.1 Shape and materials for the tests

To compare the behaviour of the materials and carving of the three Stradivari violoncello tailpieces, we decided to make three different copies reduced to the same dimensions using different materials and thicknesses thus obtaining several weights and behaviours. Eight tailpieces were produced on the same design;
Maple and African Blackwood were used to make tailpieces from a unique outline drawn numerically and cut with a numerically controlled machine (CNC).
African blackwood has a specific modulus three times higher than maple and these woods have been compared in a preceeding article [14].

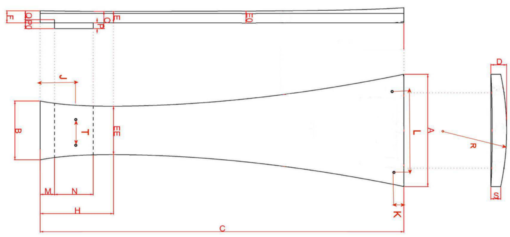

Figure 12: Same design for the eight test tailpieces produced

Eight tailpieces were made: two in plain maple with two types of attachment, Two with thin blackwood A. Blackwood adjusted shell glued on maple, two with thick A. Blackwood adjusted shell glued on maple. We will compare 2A, 5C, 7A, one with a plain African Blackwood tailpiece with two different types of attachment: 8A and 8B and a modern tailpiece made in Dyospiros spp. in H35A.

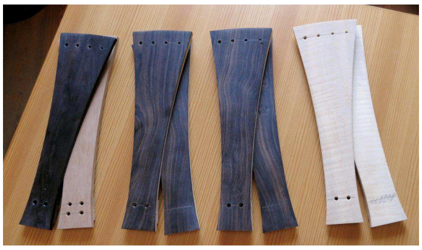

Figure 13: View from upper surface of the eight tailpieces made on the same outline. , two are in plain maple, two covered by thin 1.8 mm of A. Blackwood adjusted shell, two with thick 1.8 mm of A. Blackwood adjusted shell. One is in plain blackwood, and N°1 is in hardwood (Massaranduba) weighing 70g.

Figure 14bis: View from underneath surface of the eight tailpieces made on the same outline.

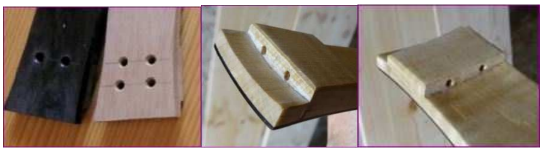

Figure 15: 2, 8A with simple attachment holes, identical square lumps with longitudinal attachment 5C, 7A and 8B.

N°	Materials	Length	Top width	Bottom width	Weight wood	Common points with tailpieces by Stradivari
N°1	Massaranduba	238	40.3	32.5	70 g	tests outline. Simple attachment
2A	Plain maple with inlays	238	40.3	32.5	49 g	E.619 material + Simple attachment
5C	Maple + fine 1.8 mm A. Blackwood adjusted shell	238	40.3	32.5	64 g	E.486.1 material + Square lump with longitudinal attachment
7A	Maple + thicker 2.8 mm A. Blackwood adjusted shell	238	40.3	32.5	78 g	E.486.2 material + Square lump with longitudinal attachment
8 A	Plain Black-wood	238	40.3	32.5	88.5 g	Same as 8B simple holes and chord
8 B	Plain Black-wood	238	40.3	32.5	88.5 g	Same square lump with longitudinal holes, metal wire
MT H35A	Dyospiros spp.	235	<<<	<<<	64 g	**Modern tailpiece**

Figure 16: Characteristics of the tested tailpieces.

The set ups of the after-length on the dead-rig are not very different from each other.
Attachments also differ, as the originals : tailpieces 2, 8A with simple attachment holes going through. Square lump with longitudinal attachment 5C, 7A and 8B.

4. METHODOLOGY

We use Stoppani's different software:
"Acquisition" is for capturing frequency response functions, using a mini impulse hammer and an accelerometer.
"ModeFit" is for estimating mode parameters and other data manipulations from the measurements.
"ModeShape" allows imposing the measured movements on the outline of the drawing, for plotting mode shapes and viewing animations and other operations on plots.
"FRFOverlay" is for comparing Frequency response functions FRFs or complex Fast Fourrier Transform FFT data and can do all sorts of averaging. [14], [15], [16], [17], [18], [19].

Eighteen points are marked on each tailpiece to place accelerometer and hammering points. For the free-free measurements, the tailpieces rest on elastic bands.

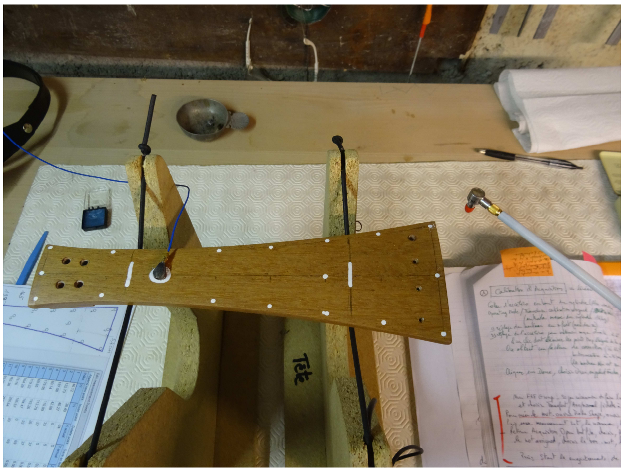

Figure 17: Free-free measurements of n°1.

For the measurements under string tension, a dead-rig is used, constructed on an IPN rail used for our preceding experiences, of standard lengths for a cello, with a solid wooden bridge, and with always the same set of modern strings, in order to keep as much as possible the same parameters each time in order to make easier comparisons on the behavior of tailpieces. The rig was tested for its inert behavior at the considered frequencies [14].

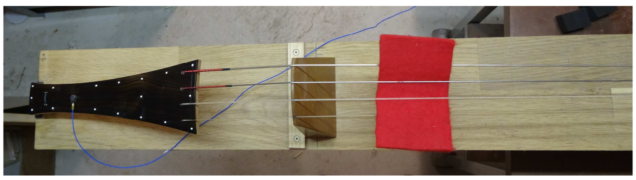

Figure 18: Tailpiece set on dead-rig for modal analysis measurements under tension of the same set of cello strings C G D A, Savarez (Middle) in bare gut for D and A and Aluminum and Silver covering on gut for G and C.

5. RESULTS

5.1 Free-free modal analysis

The reference tailpiece n°1 we show here show the modal shapes of a hardwood (Massaranduba) tailpiece of the same size, weighing 70g. A split flexion mode F1 & F2 shows similar figures at two different frequencies, but on opposite directions, as if the mechanical properties of the two sides of the piece were not symmetrical from a vibrational point of view.

We do not find this feature in the three test pieces which have a strong and unique first flexion mode F1. If our chronological conjecture is right, Strad was tuning these tailpieces higher and higher : the fundamental first Flexion mode of the thin 1.8 mm A. Blackwood adjusted shell glued on maple tailpiece 5C is one semi-tone higher than the plain maple with purfling inlays 2A, while the thick 2.8 mm A. Blackwood adjusted shell glued on maple 7A is a fourth higher the plain maple with purfling inlays 2A, being a third higher the thin 1.8 mm A. Blackwood adjusted shell glued on maple 5C. The order stays the same in higher frequencies.

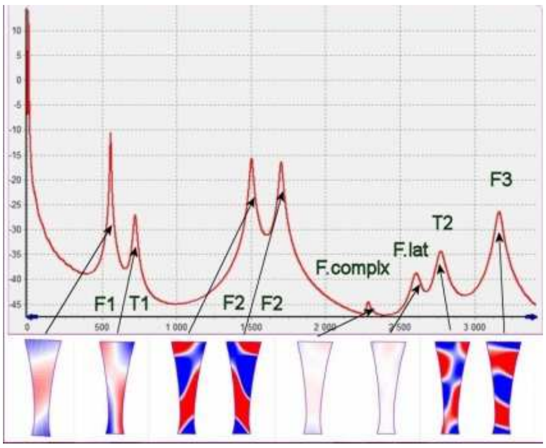

Figure 19: RMS of Tailpiece N°1, reference for free-free modes.

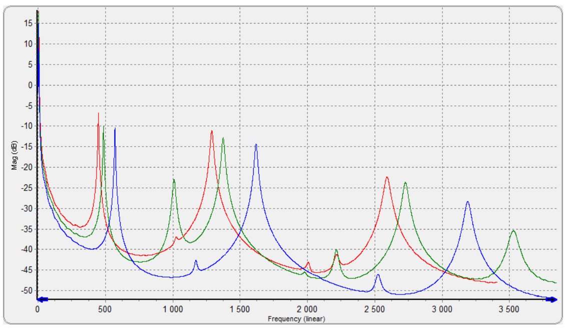

Figure 20: RMS of 3 free-free tailpieces copied from Stradivari's chosen characteristics: 2A red , 5C green, 7A blue.

This gives quite a drastic change to the main sound of the piece. If we chose a'=450 Hz, which could exist at the time (as tunings were not as fixed as today) the tailpiece 2A is tuned a', 5C is tuned b' and 7A is tuned at d". Of course, the original tailpieces are probably not exactly at these frequencies, but these measurements allow us to have an order of magnitude of these pieces made by the same maker within around some 15 years or 20 years of drastic changes of the violoncello.

Another important feature of the thin 1.8 mm A. Blackwood adjusted shell glued on maple 5C's free motion is a strong torsion mode around 1010 Hz (between b" and c3) while plain maple tailpiece 2A and thick 2.8 mm A. Blackwood adjusted shell glued on 7A, considered as the oldest and the more recent, are not a all mobile at the first torsion mode, which was not necessarely to be expected.

The second flexion mode F2 is strong in amplitude and in the same frequency order as F1 : 2A, 5C and 7A. E3, f3 and a3 : a semi-tone and a third again.

The torsion modes are negligible, and the two first flexion modes are strong.

N°	Mode 1 F1	Mode 2 T1	Mode 3 F2	Mode 4	Mode 5	Mode 6 F3	Mode 7
2A	450 Hz	1030 mini	1290 high	2010 mini	2220 mini	2590 high	
5C	480 Hz	1010 medium	1380 high	1980 mini	2210 mini	2730 high	3530
7A	570 Hz	1170 mini	1610 high		2520	3190 high	
H35A plain Dyospiros spp. Modern tailpiece	475	-	1300	1818 mini	-	2340	mini

Figure 21: Free-free frequency modes of 2A 5C and 7A test-tailpieces, compared with a modern plain Blackwood tailpiece.

The modern taipice doesn't seem to show the first torsion mode T1 between the two first flexion modes F1 and F2 : the torsion seems to have been eliminated from the lower frequencies, thus saving energy of a motion that is not useful to the bridge rotation, i.e. for the transmission of energy to the body of the instrument.

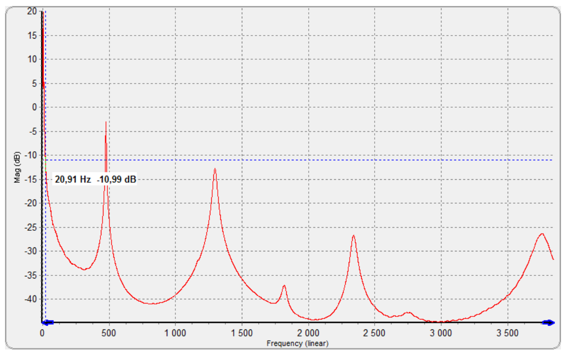

Figure 22: RMS of a free-free modern tailpiece.

This could be an element in enhancing the power of the instrument. The modes F1 and F2 are much lower on the modern tailpiece, thus enhancing the motion of the bridge in the lower notes, but may be not so efficiently if this motion is not really transmitted efficiently to the bridge itself. But the very tense steel strings probably help to do that, compare to the old gut strings used in baroque times.

To put it in fewer words, in the free-free modes, the modern tailpiece behaves more like a beam, which allowed the 1st and second flexion modes to be lowered a little in frequency, while the baroque tailpieces behave like thick plates and have twisting modes in lower frequencies.

5.2 Modal analysis of tailpieces under tension

Once under tension, obviously, the modal analysis changes dramatically from the free-free behaviour and presents a different frequency distribution, which can be interpreted in terms of mobility and energy. It is somewhat comparable to the condition under which the tailpiece will behave on an instrument, except that the acoustics of the box of the instrument itself and the action of

bow, bridge, and strings are eliminated. We had checked that the main frequencies are much higher than the ones we study here.
Under string tension on a dead-rig with a solid bridge, we showed [22] that the two first modes are solid body modes: the first one is the bridge jumping up and down. It has very strong amplitude on our RMS because we are tapping the tailpiece from the top, and it depends a lot from the length of the attachment. On a playing cello, this mode won't be as much driven by the lateral action of the bow. The second mode is the windscreen movement of the tailpiece rotating from left to right also affected by the attachment. Higher from these, we look at the flexion and torsion modes which are characteristics of the tailpieces themselves.
The third mode is the First flexion mode for all tailpieces studied here. The main differences we can see between Strad's three models of tailpieces under tension (plain maple, thin 1.8 A. Blackwood adjusted shell glued on and thick 2.8 mm A. Blackwood adjusted shell glued on maple) is to be noticed on the 7A tailpiece with thick 2.8 mm A. Blackwood adjusted shell glued on maple which manages to be higher in Frequency for F1, F2 than the same modes of the two other tailpieces (2A in plain maple and 5C with fine 1.8 mm A. Blackwood adjusted shell glued on).

N° **F (Hz)**	**Mode 1 CS**	Mode 2 CS	**Mode 3 F1**	Mode 4	Mode 5	**Mode 6 F2**	Mode 7
2A	65 Hz	178 high	490 middle	816 mini	909 mini	1304 small	1420 high
5C	**63-65 split high**	**184 high**	**522 middle**	603 mini	995 mini	1237 mini	**1400 high**
7A	**60- 97 Split high/small**	30 …370 Split High/middle	**699 middle**		2520	**1696 high**	
8A plain black wood	**56**	149	**332 /384**	703	989	**1029**	1154
8 B plain black wood	**57**	223	**535**	722 / 795	961	**1050**	1180
H35A plain Dyospiros spp. Modern tailpiece	**65**	253	**506-575 small**	676	-	**1080 – 1110 high**	**1168 high**

Figure 23: Frequency modes of test-tailpieces under tension on dead-rig.

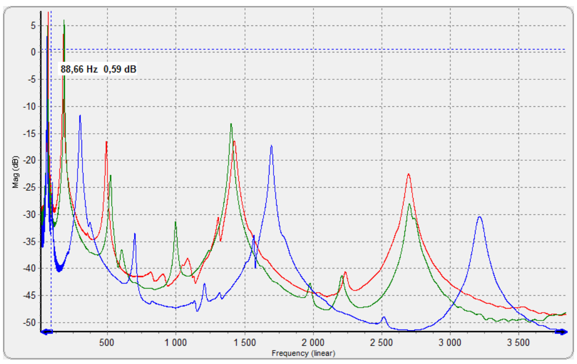

Figure 24: RMS averages of 3 tailpieces under tension on dead-rig : 2A in red, 5C in green; 7A in blue.

The two plain Blackwood tailpieces 8A and 8B differ by their attachment; the first has a chord trough to simple holes, the other longitunial holes through the lump and metal wire. We confirm the imporance of this for the Solid Body modes, and can see that it doesn't impact much the frequency of the flexion modes. Nevertheless, we can see an impact of the rigidity of the metal wire in the splitting of the modes.

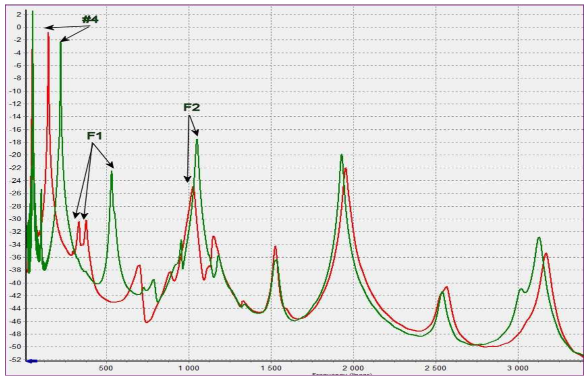

Figure 25 : RMS averages of two blackwood tailpieces with different attachments: 8A (red) simple holes and chord and 8B (green) longitudinal holes, metal wire on a dead-rig.

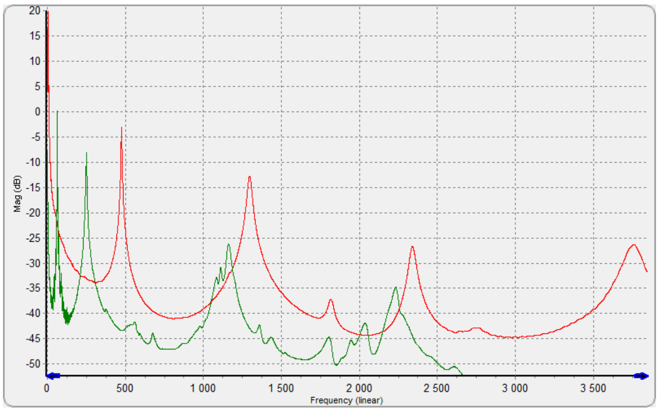

Figure 26: RMS of a modern tailpiece in free-free (red) and under tension on a dead rig (green).

The vibrations of modern tailpiece in free-free under tension on the dead-rig do'nt really have a torsion mode before the first flexion mode, feature that we have seen already on the free-free modal analysis. It shows lower amplitudes and a split of the first Flexion mode at low amplitude. It looks like there is less energy going into the tailpiece, and the pics are less clear and splitted, which indicate less define frequencies. This probalbly avoids to enhance one frequency in particular, which would create wolf notes and absorb energy when the corresponding note is played.

6. CONCLUSIONS

Stradivari has designed stronger and stronger, stiffer and stiffer, heavier an heavier tailpieces as the cello was becoming more of a solo instrument and being played more in the higher positions. Also, the attachment became stiffer as virtuosity and tessiture increased. The mobility was thus diminished, even without fixing the tailpiece like the anchorage on a viol, which could dampen the vibration of the bridge, and he instead used a metal wire allowing left to right movements. He must have been concious of the energy loss that the older very mobile tailpieces could cause to the sound of the cello, and this show how the famous maker experimented with and payed attention to each piece of the instrument in order to transfomr the cello from a concert group instrument into a powerful solo tool.

Next orientations for this research will be to measure models of tailpieces form the second half of the 18^{th} c. and of the 19^{th} c. to establish a chronology of behaviour, using appropriate stringing. Another approach will be to do a link between acoustical properties and musical perception: powerfulness, playability, musicality, and harmonic complexity.

7. REFERENCES

[1] Frederick V. Hunt, *Origin in Acoustics*, Acoustical Society of America, 1992 (Reprint), p.1.

[2] Gioseffo Zarlino, *Le istitutioni harmoniche del reverendo M. Gioseffo Zarlino […]*, Venise, s.n., 1562, part. 2, chap. 24, p. 93-96, 126.

[3] Houssay A., "Cordes filées et violons en Italie au XVIIe siècle ; quelques cas d'instruments crémonais recoupés" in "L'orchestre à cordes sous Louis XIV - Instruments, répertoires, singularités" Jean Duron & Florence Gétreau Dir., Vrin, Paris, to be published in 2015.

[4] Marin Mersenne, *Novarum observationum physico-mathematicarum*, Paris, Antoine Berthier, 1647, vol. 3, p. 165; see Barbieri, ≪ Gold and Silver ≫, p. 123.

[5] Patrizio Barbieri, ≪ Gold and Silver-Stringed Musical Instruments : Modern Physics vs Aristotelianism in the Scientific Revolution ≫, *Journal of the American Musical Instrument Society*, 36, 2010, p. 11-154.

[6] Piero Gargiulio, ≪ Strumenti musicali alla corte medicea : nuovi documenti e acognosciuti inventari (1553-1609) ≫, *Note d'archivio per la storia musicale*, 3, 1985, p. 70 ; see Barbieri, ≪ Gold and Silver ≫,p. 120.

[7] Stephen Bonta, ≪ From Violone to Violoncello: a Question of Strings ? ≫, *Journal of American Musical Instruments Society*, 3, 1977, p. 64-99.

[8] Sacconi S., Les "secrets" de Stradivari, Castlli & Bour'his, Nantes, 1989.

[9] Faber, Tony, Stradivarius: Five violins, one cello and a genius, Pan Macmillan, 2011, p.55-56.

[10] Chouquet, G., "Le Musée du Conservatoire national de musique – Catalogue raisonné des instruments de cette collection", Paris, 1875.

[11] Houssay Anne, Fouilhé Eric, "The string "after length" and the cello tailpiece: History, acoustics and perception", International symposium of Musical acoustics: ISMA'2014, Le Mans, July 7-12 2014.

[12] Fouilhe E., Houssay A., Brémeaud I.,"Dense and hard woods in Musical instrument making: comparison of mechanical properties and perceptual quality grading", Proceedings of ISMA'2014, Nantes 2012, p. 2689-94.

[13] Houssay A., Fouilhé E., "The string "after length" and the cello tailpiece: History, acoustics and perception", - Fouilhé Eric, Houssay Anne, "String "after-length" and the cello tailpiece: acoustics and perception", Stockholm Music Acoustics Conference SMAC 2013.

[14] Hutchins, C. M., "The effect of relating the tailpiece frequency to that of other violin modes", Catgut Acoustical Society Journal, Vol.2 N°3, (Series II), 5-8, 1993.

[15] Stough, Bruce, "The Lower Violin Tailpiece Resonances", Catgut Acoustical Society Journal, Vol. 3 (Series II) May1996, p. 17-24.

[16] Woodhouse, J., "Body vibration of the violin- What can a maker expect to control ?", CAS 10. J Catgut acoustical society journal, vol 4 N°5 serie I, 2002.

[17] Fouilhe Eric, Goli Giacomo, Houssay Anne, Stoppani George, "Preliminary study on the vibrational behaviour of tailpieces in stringed instruments", COST IE0601, STSM results, Hambourg, 2009.

[18] E. Fouilhe, G. Goli, A. Houssay, G. Stoppani, "The Cello tailpiece: how it affects the sound of the instrument", Viennatalk 2010.

[19] "Vibration modes of the cello tailpiece", in *Archives of Acoustics*, 36, 4, 2011, pp. 713-726. http://acoustics.ippt.gov.pl/ DOI: 10.2478/v10168-011-0048-2.