USING A DUAL ELECTRONIC SPECKLE-PATTERN INTERFEROMETER TO STUDY COUPLED VIBRATIONS IN DRUMHEADS

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ABSTRACT

A dual electronic speckle-pattern interferometer (ESPI) system is described that allows images of operational deflection shapes of two sides of a vibrating object to be viewed simultaneously and recorded. Experimental details of the system are discussed and applied to the study of musical drums with two heads at either end of a cylindrical shell. In particular, methods for determining the degree of coupling and the phase relations between the two oscillating heads are shown. A variable length drum has been constructed and used to investigate the coupling of membrane vibrational patterns as a function of the distance between the heads. It is shown that the coupling vs. length data depend strongly on the shapes of the vibrational patterns. These coupling trends are illustrated and further interpreted with the use of a finite element model of the drum, which shows the role of the enclosed air motion within the shell.

1. INTRODUCTION

Many musical instruments exhibit vibrating surfaces on opposite sides of an air cavity (e.g. violins and guitars). These surface vibrations may be coupled to varying degrees by the enclosed air and the supporting mechanical structures. Drums containing membranes at opposite ends of a cylindrical shell (e.g. tom toms, snare drums, and bass drums) provide particularly good examples of this type of coupling in the context of a relatively simple geometry (Figure 1).

Figure 1. Tom toms (left) and snare drum (right) contain circular membranes at opposite ends of a cylindrical shell.

The first four normal mode shapes of a single ideal membrane are shown in Figure 2. The modes are identified by the number of nodal diameters and circles (*m*,*n*) respectively. Modes containing at least one nodal diameter are doubly degenerate, with two orthogonal orientations having the same frequency [1]. In real drums, slightly non-uniform tension in a head may lift this degeneracy resulting in closely spaced frequency pairs [2]. Non-uniform tension can also act to distort the symmetry of the nodal diameters and circles predicted for the ideal membrane.

In the coupled drum system, both membranes exhibit operating deflection shapes that are linear combinations of the ideal membrane modes, and which can still be adequately identified by the number of nodal diameters and circles, at least for the lower frequency resonances.

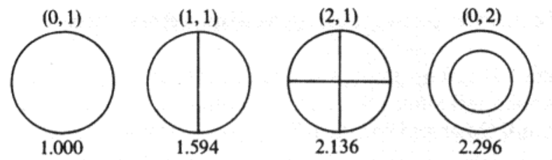

Figure 2. The first four normal mode shapes of an ideal membrane, with relative frequencies indicated below. The integers (*m*,*n*) designate the number of nodal diameters and circles respectively.

When one side of a drum is struck (the “batter” head), both sides vibrate. The coupling of the two heads is provided acoustically by the enclosed air and mechanically by the shell [3, 4]. In the current work, only the coupling due to the enclosed air is considered. The degree of coupling of the vibrational patterns on the two membranes depends on several factors including drum geometry (diameter and length), head materials, and tensions. With identical heads (as used in the experiments described below) strongly coupled shapes will have similar vibrational amplitudes when driven at a resonant frequency. Weakly coupled cases will result in an amplitude ratio that deviates from unity. “Uncoupled” cases have essentially no detectable motion on one head, while the other displays a resonant pattern. The degree of coupling also varies with the shapes of the vibrational patterns and the relative phase relation between the two heads. The lowest frequency (fundamental) mode always couples strongly, displaying the (0,1) shape, with both heads moving in phase [3]. Higher frequency resonances exhibit varying degrees of coupling, with membrane motions that are either in, or out of, phase with each other.

An optical system has been built to create interferometric images of both vibrating drumheads simultaneously. When the drum is excited acoustically at a resonant frequency, the operating deflection shape of each head can be recorded along with amplitude and phase information as described in Section 2 below.

2. EXPERIMENTAL SETUP

The optical system is based on the electronic speckle-pattern interferometer (ESPI) shown in Figure 3 and described in Reference 5. Light from a helium-neon laser is split into reference and object beams, which are then expanded using microscope objectives (not shown). The object beam reflects off the vibrating surface, while the reference beam travels a similar distance without encountering the object. The beams are recombined at the CCD camera and the resulting signal is processed [6] to create speckle-pattern interference images in real time on the computer monitor. When the object is driven acoustically at a resonant frequency, the ESPI images show operating deflection shapes with nodal lines in white. Gray fringes indicate contours of equal amplitude motion normal to the surface of the object.

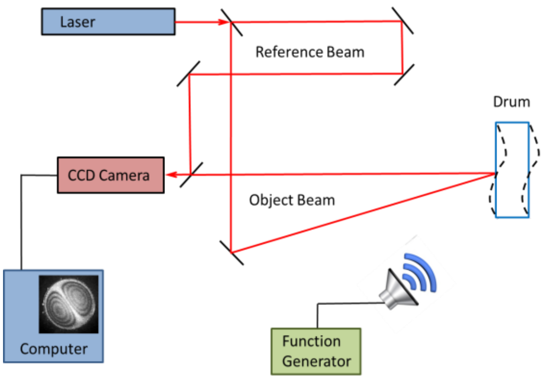

Figure 3: Schematic diagram of the electronic speckle-pattern interferometer (ESPI) used to image one vibrating surface. In the resulting images nodal lines appear white; gray fringes represent equal amplitude contours.

2.1 Dual ESPI system

In order to record images of both heads of a drum simultaneously, a dual ESPI system was constructed by adding a second, identical, setup directed at the opposite side of the drum as shown in Figure 4. With this arrangement, coupling of the drumhead vibrations can be observed. In particular, the operating deflection shapes and orientations can be compared in real time. In addition, the gray contour fringes can be counted to provide an estimate of the relative amplitudes of the two heads. It should be noted that in the case of coupled oscillators the *relative* amplitude does not depend on the location of the driving speaker. In practice, the speaker location is chosen to provide maximum clarity of the fringes.

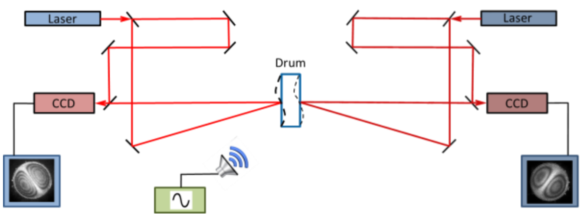

Figure 4: Dual ESPI system showing speckle-pattern images of both drumheads.

Due to the fact that the two CCD cameras view the drum from opposite directions, one image must be flipped horizontally before the relative orientation of the two images can be judged. In the lab this is achieved in real time by viewing one of the two computer monitors in a mirror (Figure 5). After the images are recorded, one is rotated digitally prior to presentation. Thus the images shown in Figure 4 will be viewed as in Figure 5, with alignment clearly visible in this particular case.

Drumhead resonant frequencies are initially determined using a spectrum analyzer with a microphone input as the drum is struck. Coupled vibrational patterns are then identified by scanning the acoustic driving frequency near these resonances while observing the ESPI images on both monitors (one via the mirror). Coupled patterns show very similar shapes and orientations, and both patterns reach their maximum amplitudes at the same frequency. The strength of the coupling may be qualitatively determined from the amplitude ratio, which is obtained by counting fringes in equivalent regions of the images.

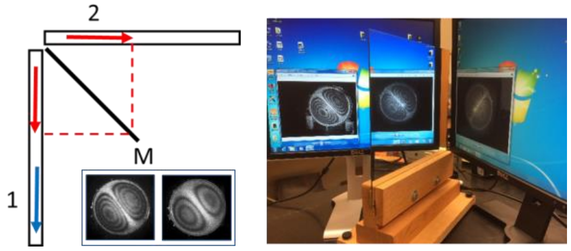

Figure 5. Computer monitors 1 and 2 display ESPI images from opposite heads of a drum. Mirror M is used to reverse the image from monitor 2 so the relative orientations can be viewed in real time.

2.2 Phase determination

As described above, The ESPI system produces white regions in the images where the object beam is unchanged during the drum head oscillation, i.e. at the nodal lines. However, these image pairs by themselves (e.g. Figure 5) do not provide phase information. Corresponding regions of the coupled vibrations shown in Figure 5 must be either in, or out of, phase with each other [7, 8]; this relative phase can determined by adding a moving mirror into the object beams of both sides of the system as described below.

A mirror in the object beam is attached to a mechanical oscillator [9] that is powered by the same function generator that drives the speaker (Figure 6 shows one side of the system). Thus, the mirror oscillates at the same frequency as the drumhead. In this configuration, nodal regions of the membrane no longer appear white. Instead, portions of the drumhead moving with the same amplitude and phase as the mirror now appear white. Because drumhead motion exhibits opposite phases on either side of a nodal line, the white line in the image will shift to one side or the other when the mirror motion is activated. This shift of the white regions, applied to both sides of the system, can be used to determine the relative phase of the two sides of the drum, and the ambiguity of phase in Figure 5 can be removed.

A decade resistance box is used in series with the mechanical oscillator to control the amplitude of the mirror motion, so that it can be adjusted to match a portion of the drumhead motion. A double pole double throw knife switch is used to reverse the phase of the mechanical oscillator as needed. Thus, each mirror can oscillate either in, or out of, phase with the driving speaker (and with the other mirror). The phases of the mirrors relative to each other can be verified by direct visual observation of their motion at low frequency (e.g. a 1 Hz square wave) and large amplitude.

However, in general no portion of the drumhead will be in phase with the driving speaker due to the time of flight of sound from the speaker to the drumhead. In such cases no white region will be seen in the images. The phase of the drumhead relative to the speaker cone is easily adjusted by varying the speaker location (i.e. its distance from the membrane.)

The appropriate speaker distances, which vary with driving frequency, are integer multiples of acoustic half wavelengths and can be roughly estimated assuming straight line motion of sound from speaker to drumhead. However, in practice it is easiest to simply move the speaker by hand with the mirror motion activated, while watching the monitor for a clear signature (i.e. a white line that clearly deviates from the stationary mirror nodal line.) When this signature is observed, the amplitude of the mirror motion can be adjusted using the resistance box in order to maximize the clarity of the image. Reversing the mirror phase provides a way to double check the initial interpretation of the image, as the white line should now shift in the opposite direction.

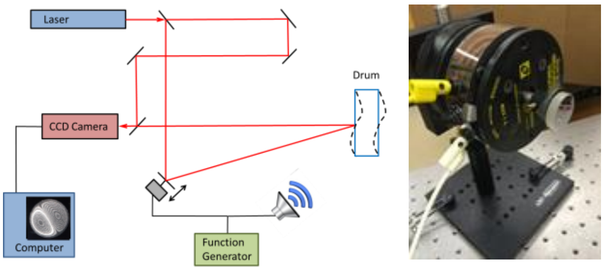

Figure 6. Moving mirror in object beam indicated with arrow (left). Mechanical oscillator (right) is driven from the same signal as the speaker that excites the drumhead.

Figure 7 shows three representative ESPI images of a single drumhead driven at a (2,1) resonance. The two nodal diameters and the nodal perimeter circle are visible in the first image, taken with no mirror motion. Viewing the nodal diameters as roughly vertical and horizontal, one expects motion in quadrants one and three to come toward the viewer when quadrants two and four move away (and vice versa). The next two images show the white regions shifted due to mirror motion of opposite phases, in agreement with this assumption. By observing similar images for both sides of the drum, and knowing the relative phases of the mirrors themselves, the relative phase of the drumhead motion can be determined unambiguosly.

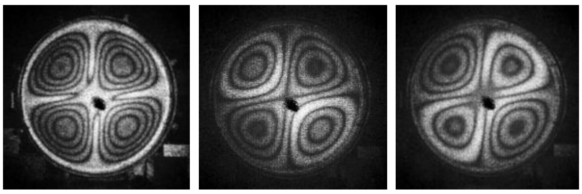

Figure 7. ESPI image of a (2,1) resonance pattern on a single drumhead (left). The same pattern with the mirror motion engaged (center and right). Mirror phase is reversed between center and right images. Note that the perimeter, which is always stationary, is not white in the moving mirror images.

3. FINITE ELEMENT MODEL

A finite element model of the drum, including the two heads and the enclosed air, was developed using commercial software [10]. The shell, which can play a role in the coupling process [3, 4], and the external air were not modelled. The model was used for qualitative comparison with experimental results and to explore the parameter space in terms of drum sizes and membrane tensions to investigate in the lab. Most importantly, the model provided insight as to the probable behavior of the air column inside the shell and its role in the coupling process.

Inputs to the model include drum geometry (diameter and length), head material properties (density, Young's modulus, and Poisson's ratio) and the membrane tensions. For this work identical membrane materials were used. In most cases the tensions in the two heads were set to differ from one another by a few percent. The model produced the following outputs: normal mode frequencies and shapes (including relative orientations), relative amplitudes, and relative phases for both heads and the enclosed air column. Visual representations of the mode shapes appear as in Figure 8, with nodal lines and planes depicted in green.

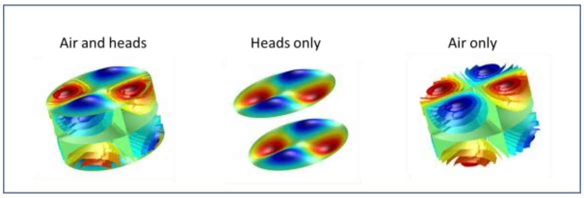

Figure 8. Representative images from the finite element model showing normal mode shapes for the two membranes and the enclosed air column. Nodal lines and planes are represented in pale green. Membrane images show displacements; air images show planes of equal pressure.

4. DRUM EXPERIMENTS

4.1 Variable length drums

Two variable length drums were constructed (Figure 9) allowing the heads and their tensions to remain fixed while coupling vs. length experiments were performed.

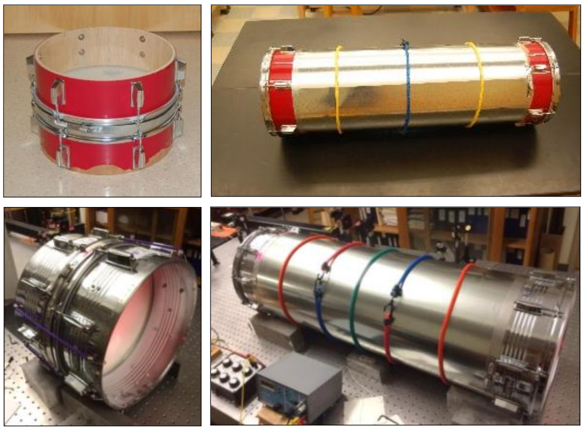

Figure 9. Variable length drums with shells replaced by sheet metal. 12" diameter tom tom (top) and 14" diameter snare drums (bottom). Lengths vary from about 2 to 110 cm. Heads face each other (left) for very small separations.

The first drum constructed was a 12" diameter tom tom cut in half to produce two shorter shells, each with a single head. The two sections were then connected using various lengths of sheet metal formed into a cylindrical shape and attached with modified hose clamps. The second variable length drum consisted of two 14" diameter snare drums, each with the snares and corresponding head removed. Again, these two drums were connected to sheet metal cylinders of various lengths, up to a maximum length of just over one meter. These drums could also be connected with the heads facing each other for very short lengths, as small as 2 cm. Identical single-ply Mylar heads [11] were used in all cases.

Drumhead tensions were adjusted via the standard tuning lugs spaced evenly around the perimeter (see Figure 1). The membrane tensions were tested for uniformity and for overall pitch using a commercially available mechanical gauge [12] that is placed directly on the head. Prior to assembly of the full drum, pitch and tension uniformity were also verified using a spectrum analyzer with a microphone input.

4.2 Experimental results

Several coupling trends were identified in the lab using the dual ESPI system with the variable length drums. These trends were qualitatively consistent with the predictions of the finite element model, including the shapes, amplitude ratios, and phase relations in the cases that were clearly coupled.

4.2.1 Coupling vs. length

The coupling strength of the vibrational patterns as a function of drum length *L* varies with shape. For patterns including only nodal diameters (in addition to the single nodal circle at the perimeter) the coupling strength decreases rapidly with drum length. These "diameter shapes" include (1,1), (2,1), and (3,1), etc. patterns. The finite element model suggests this behavior is due to air motion that "sloshes" back and forth azimuthally and can therefore be localized near a single head, as described by Rossing *et al*. [3, 4].

Figure 10 shows experimental ESPI images of (1,1) shapes on both heads of a drum with lengths varying from 2 cm to 55 cm. The fringe ratio begins at approximately one with the heads very close together and deviates from unity as the length is increased. At a length of 55 cm no vibrational pattern is visible on the head shown on the left. The resonant frequencies (not shown) also decrease moderately with increasing separation.

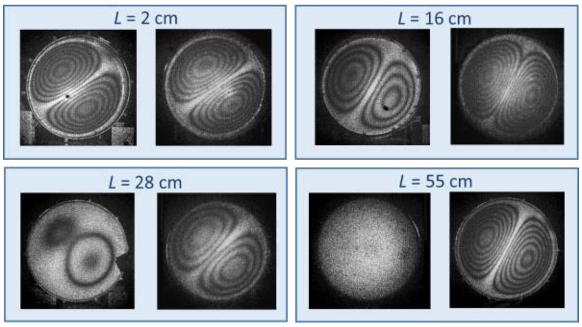

Figure 10. Coupling of (1,1) shapes vs. drum length. Amplitude ratios are inferred from the relative number of fringes on each head. The ESPI images show coupling strength varying from strong, at *L* = 2 cm, to barely visible at *L* = 28 cm. At *L* = 55 cm no coupling is observed.

The finite element model also shows how this (1,1) trend differs from that of the (0,1) fundamental (Figure 11). Shapes containing only circular nodal lines couple with longitudinal air motion, resulting in coupling between the heads over much larger lengths. Unlike the diameter shapes, these "circle shapes" do not conserve air volume and require some degree of longitudinal air motion. In particular, the fundamental (0,1) shape, with heads in phase, couples strongly for all lengths studied in the lab (up to 110 cm) as well as for all corresponding cases examined numerically with the finite element model.

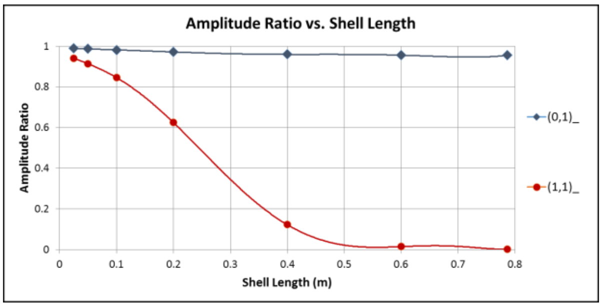

Figure 11. Finite element predictions for coupling of (1,1) shapes (red) and (0,1) fundamental shapes (blue) as a function of drum length. The (0,1) shapes couple strongly at all lengths via longitudinal air motion.

4.2.2 Diameter shapes vs. m

At a fixed length, the coupling strength of "diameter shapes" decreases with the number of diameters, *m*. Experimental ESPI images for the first four *m* values of a drum of length *L* = 6 cm are shown in Figure 12. The amplitude ratios, as determined by the relative numbers of contour fringes, clearly show the coupling strength decreasing as *m* varies from one to four. These results are qualitatively consistent with the finite element model, which is used to plot similar predictions in Figure 13.

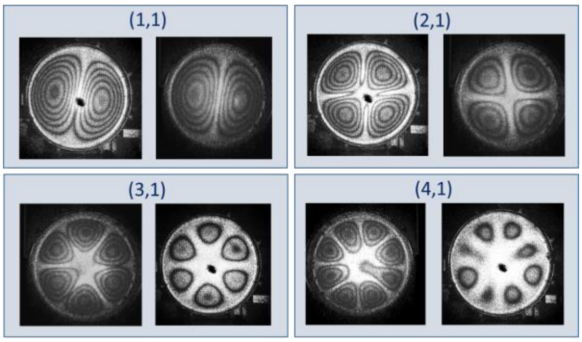

Figure 12. ESPI images showing coupling of "diameter shapes" at a fixed length of *L* = 6 cm. Equal amplitude fringes show the coupling strength decreasing as the number of nodal diameters (*m*) increases from 1 to 4.

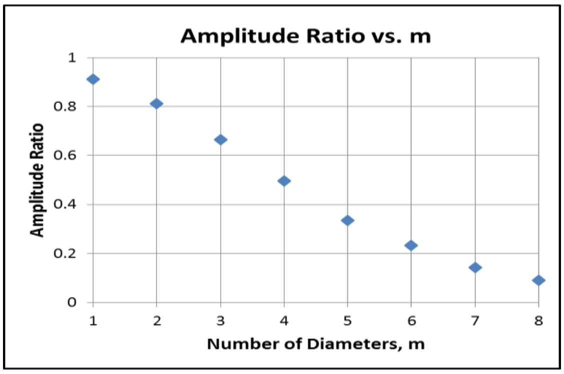

Figure 13. Amplitude ratio vs. number of nodal diameters at fixed drum length, calculated from the finite element model. Strongest coupling (amplitude ratio near unity) occurs with one diameter (1,1) and decreases steadily with increasing *m*.

5. CONCLUSIONS

A dual ESPI system has been constructed enabling opposite heads of a drum to be imaged simultaneously. Various aspects of the coupled vibrations can be determined including the approximate degree of coupling, relative orientation of the operating deflection shapes, and relative phase. A finite element model has been created using commercial software that agrees qualitatively with the experimental results and provides plausible representations of the air motion inside the drum shell. Variable length drums have been constructed and used to investigate membrane coupling trends as function of shell length.

Qualitative membrane coupling trends have been identified based on the characteristic shapes having either nodal diameters or nodal circles on the membranes. “Combination shapes” containing both nodal diameters and circles are also observed in the lab but are not addressed in this paper. Diameter shapes generally couple strongly only over small lengths and with small numbers of diameters. This may be explained by a localized “sloshing” motion of the enclosed air that need not extend from one head to the other. “Circle shapes,” without nodal diameters, do not conserve volume and appear to couple with longitudinal air motion. Thus, these patterns couple over the full range of lengths investigated.

Acknowledgments

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6. REFERENCES

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