SOFTWARE SIMULATION OF CLARINET REED VIBRATIONS

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ABSTRACT

Electric Circuit Analysis Programs, such as MicroCAP [1] are useful for simulating acoustical and mechanical behaviour of musical instruments. As has been shown in [2], frequency dependent characteristics such as acoustical impedances of wind instruments can be simulated. Also pressure and volume-flow inside a tube can be demonstrated graphically [3]. A so called AC-analysis was used for these tasks.

In the present paper first the transient response of a purely mechanical device, the clarinet reed, is studied. MicroCAP offers the possibility to show several parameters on a time scale. For this a different analysis is used, namely TR-analysis (transient analysis).

The quasi-static relation between volume flow and pressure difference is the only acoustical-mechanical question that is dealt with in this paper.

The paper explains the electro-mechanical-acoustical analogies that are the basis for the simulations. Finally the suitability of the software-model used is demonstrated by checking the results against the literature [4], [5].

1. INTRODUCTION

1.1 The software model

The model of the reed used here is the most simple one. It consists only of lumped elements. As has already been shown [5], lumped elements are useful for understanding the basic properties of clarinet reeds.

In section 2 we compare the results of Walstijn and Avanzini [4], [5] with those from the software-models used here. Conclusions are given in section 3 and an outlook to future work in section 4.

1.2 Electro-mechanical analogies

The partial differential equations that describe electrical processes are very similar in many cases to those for describing mechanical and acoustical processes. Therefore using suitable mapping, mechanics and acoustics can be represented by electrical circuits. The analogy used here can be described as follows: The three main passive components of electricity *L, C, R* represent in mechanics mass, compliance and friction. More details and tables are given in the appendix.

2. COMPARISON WITH THE RESULTS OF WALSTIJN AND AVANZINI

2.1 Motivation

Walstijn and Avanzini [4] and [5] have studied the applicability of numerical simulations to clarinet reeds. Whereas [4] deals with a one-dimensional distributed model, [5] considers a lumped model (see Fig. 1 in [5], repeated here as Fig. 1). We keep to the symbols used by Walstijn and Avanzini if not otherwise mentioned.

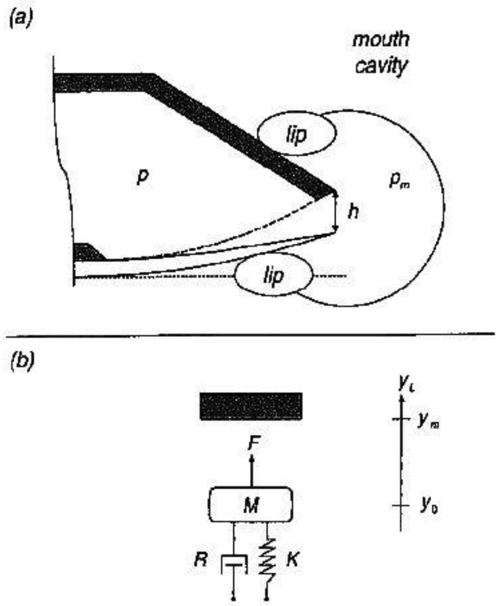

Figure 1. (a): Schematised view of a clarinet reed-mouthpiece-lip system. The dashed line indicates the profile of the mouthpiece lay, and p and p_m denote the mouthpiece pressure and the mouth pressure, respectively. (b): One-mass model of the reed tip vibration, with effective mass M, effective damping R, and stiffness K. The effective external force F exerted on the reed equals $S_d \Delta p$, where S_d is the effective driving surface, and $\Delta p = p_m - p$ is the pressure difference across the reed. The reed opening h is related to the reed tip position y_L by $h = y_m - y_L$, where y_m is the vertical position of the mouthpiece lay tip.

Fig. 1, The lumped model (Fig. 1 of [5])

2.2 Reed resonance peaks

These above mentioned two papers are good entry points for studying the applications of electronic Circuit Analysis Programs. First the excursion response of Fig. 2 of [5] is reconstructed. The reed model for this is shown in Fig. 2. Examples of typical values used are shown in the list of definitions. In MicroCAP the units are not given explicitly. “1m” does not mean 1 metre but 1 milli. SI-units are taken for granted. Depending on the physical dimension meant, 1m can stand for 1 mm, 1 mV or 1mA, etc.

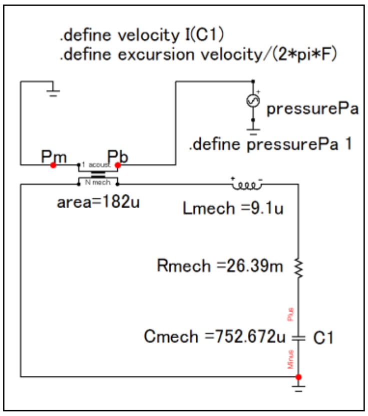

Fig. 2, Model used to reconstruct Fig. 2 of [5]

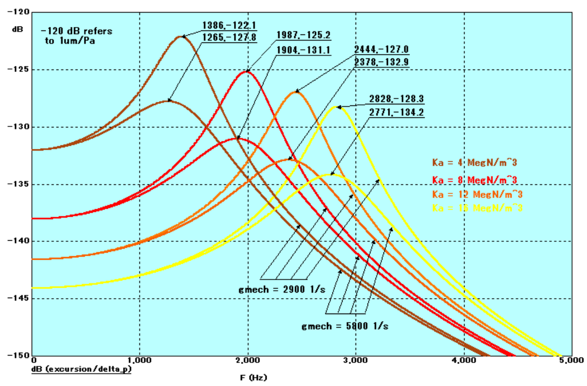

Fig. 3, Reconstruction of Fig. 2 in [5] with the original damping factor g_{mech}=2900, as well as doubled 5800

The following definitions show the syntax of MicroCAP for the physical constants and variables as well as the values used for the reproduction of Fig. 2 in [5].

.define delta_p v(Pm)-v(Pb)
.define R1value 1

.define width 13m	Table I of [4]
.define length 34m	Table I of [4]
.define area length*width	

.define Ma 0.05	Table I of [4]

.define Lmech Ma*area

.define Ka 4Meg (stepped to 8, 12, 16 Meg)

.define Cmech 1/Ka*area

.define gmech 2900	3.2 of [5]
.define Ra gmech*Ma	3.2 of [5]
.define Rmech Ra*area	

.define excursion i(C1)/s

The symbol *s* in the excursion definition stands for $j\omega$ in MicroCAP. The result is independent of the choice of R1. The signal generator delivers a cosine voltage. The result is also independent of the amplitude. During the AC-analysis the generator is swept from 0 to 5000 Hz. The lower curves of Fig. 3 show the excursion per *delta_p* for an increased value of g_{mech} of 5800/s. The reed length was chosen according to [4] Table I. The value of the area has no influence on the result of Fig. 2 in [5]. We see here that the peak is not the same as resonance frequency. They are only the same for symmetrical peaks (achieved by multiplying by frequency or plotting velocity instead of excursion).

2.3 Response comparisons

Fig. 7 of [5] shows the response of the freely resonating reed to a short Hanning pulse. The reconstruction (Fig.4) for the lumped model gives a similar result. The model for Fig. 4 is shown in Fig. 5.

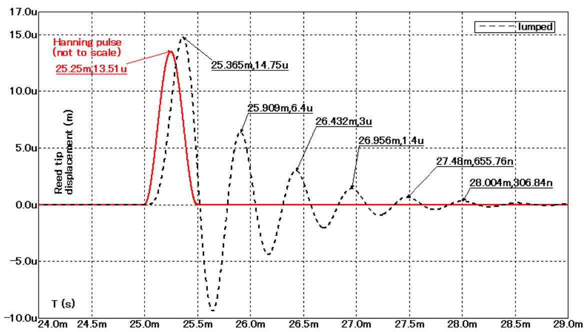

Fig. 4, Response of the reed to a short pulse

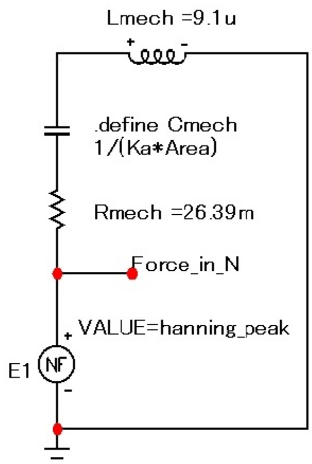

Fig. 5, Model to reconstruct the transient response.

A Hanning pulse of 0.5 ms was used. Its peak value was chosen in such a way that the maximum excursion equals the one in Fig. 7 of [5]. The radian frequency of the exponentially decreasing reed oscillation ω_r corresponds to equation (11) in [5] - (ω_r^2 is a printing error, also in (10) of [5]). The corrected equation (11) is shown here as (1).

$$\omega_r\left\{\Delta p(n)\right\} = \sqrt{\frac{K_a\left\{\Delta p(n)\right\}}{M_a} - \frac{g^2}{4}} \qquad (1)$$

All this is in the small-signal range where the nonlinear model would be inappropriate.

2.4 Reed tip displacement

Here we compare the results of section 4.1 in [5] with those obtained using MicroCAP. The part d of Fig. 6 of [5] is recalculated in Fig. 6 of the present paper. The model is shown in Fig. 7. It is similar to Fig. 2, except that a DC-offset according to (24) of [5] is added, and the condenser is nonlinear. The vertical axis here corresponds to Fig. 8 of [4], with rest position of the reed at 0.85mm and hard-limit at 1.25mm, which will never be reached in the static state.

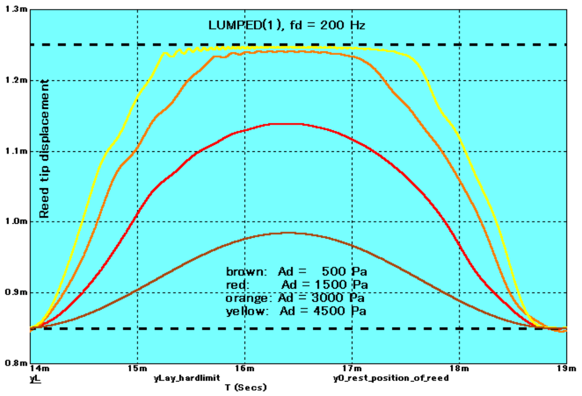

Fig. 6, Reed tip displacement as in [5], Fig. 6d

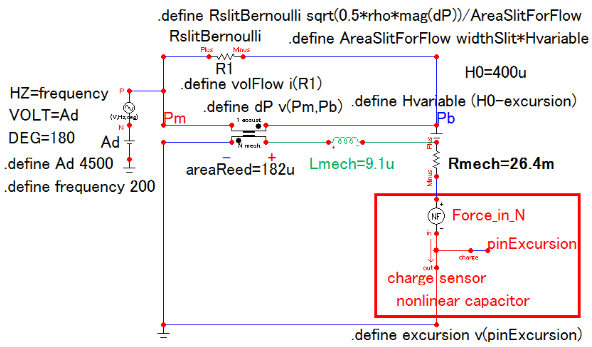

Fig. 7, The model used for the simulation in Fig. 6

The resistor *RslitBernoulli* is responsible for the pressure difference ΔP between the mouth (P_m) and the bore (P_b) of the clarinet. The simple model, without refinements, depends entirely on the Bernoulli effect $\Delta P=1/2\ rho\ v^2$ to throttle the air flow, viscosity playing no role. Since the B. effect is essentially an expression of energy conservation in the air flowing, no viscosity effects in the narrow aperture are allowed, at least not while the aperture is open. Otherwise there would be energy exchange with the walls: contradicting the key mechanism for flow regulation.

The nonlinear characteristic used as input for these nonlinear simulations is shown in Fig. 8. Although the vertical axis represents force, it is expressed in Pa to ease comparison with Fig. 8a of [4] on which it was modelled.

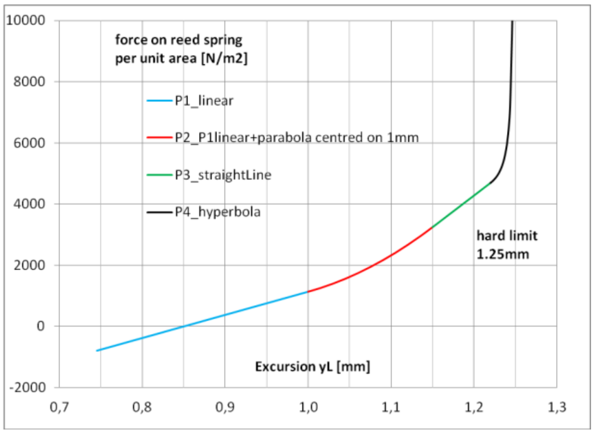

Fig. 8, The curve used for nonlinear simulations in the present paper. Physically the vertical axis P_{nonlin} really represents a force $F_{nonlin} = P_{nonlin} \times areaReed$.

Of course pressure is not an appropriate input parameter for describing a mechanical spring such as a reed, whether linear or nonlinear. We need that part of the force responsible for stretching the spring, whereas part of it is used for overcoming inertia and friction. The parameter actually used in the simulations was F_{nonlin} derived from static P_{nonlin} taking into account the effective area. In the simulation the force at any moment stretching the spring was deduced from the excursion, as shown in the circuit (Fig. 7).

In Fig. 9 we see the curve for K_a. It cannot be used reliably as input for a simulation. But it can readily be derived from Fig. 8 dividing P_{nonlin} [Pa] by *excursion* [m].

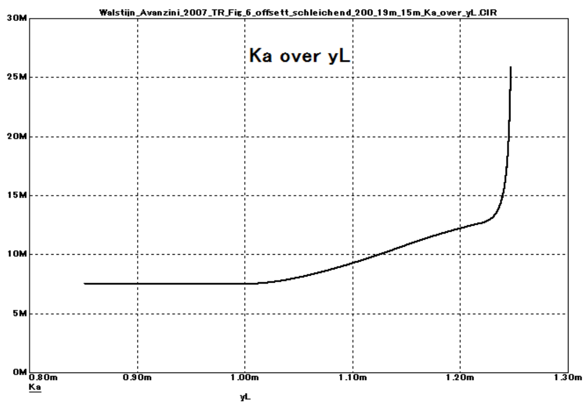

Fig. 9, K_a versus yL corresponding to Fig. 8

It is important to be aware that K_a is defined as depending on yL-$0.85mm$ in the dynamic simulation of Fig. 6. But K_a itself as a function of Δp is only useful for static simulations. Because in the dynamic state yL not only depends on Δp but also on the momentary inertial and resistive forces of the reed mass L_{mech} and the resistance R_{mech} respectively.

2.5 Volume flow versus pressure difference

In [5] (section 5.2.1 and Fig. 8) the relation between volume flow and pressure difference is treated. There is some limit in the validity of (31) of [5]. Because for a positive volume flow u_f for $\Delta p>0$ it is required that $K_a > \Delta p/(ym - y0)$. The equation (31) is reproduced here as (2).

$$u_f = w\left[y_m - y_0 - \frac{\Delta p}{K_a(\Delta p)}\right]\sqrt{\frac{2\Delta p}{\rho}} \qquad (2)$$

Fig. 10 shows u_f versus $\Delta p>0$ using the force corresponding to Δp vs. yL of Fig. 8 in [4]. The result is similar to that of Fig. 8, "lumped (1)" in [5]. Additionally the effect of a softer reed is shown. The reed never closes completely due to the non-linearity which contains a hyperbolic function.

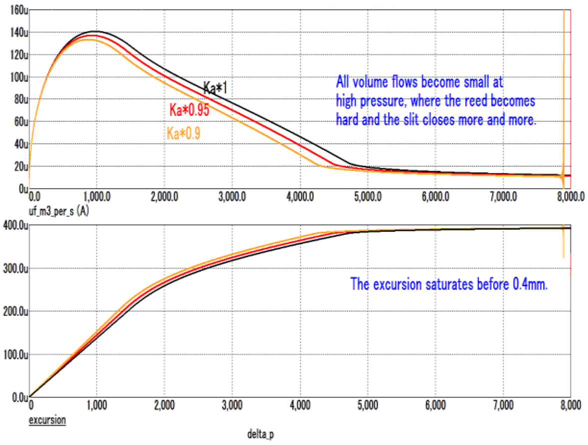

Fig. 10 Volume flow and excursion vs. pressure difference

3. CONCLUSION

It could be shown that a Circuit Analysis Program is well suited for studying mechanical and acoustical properties of a single reed generator for self sustained oscillations, as used e.g. in a clarinet. AC-analysis for frequency response of several parameters and TR-analysis for time dependent processes can be built up easily together with the needed models.

4. FUTURE WORK

That which was described here is just the beginning of the work on a refined model of the clarinet reed oscillator. It is used for testing of the reed part and the tools. One of the refinements is a distributed reed model similar to the one in [4]. This will then be coupled to a resonance tube. We intend to find out e.g. how many reed elements are needed to get a realistic simulation. This will include the behaviour of the reed generator depending on the distribution of stiffness, mass and resistance of the reed, the curve of the mouthpiece lay and the lip force and damping. A further goal is to develop a library of macros (elementary models) for the different parts of a clarinet or other woodwind instruments.

5. APPENDIX: UNAMBIGUOUS ELECTRO-MECHANICAL-ACOUSTICAL ANALOGIES

There are 3 domains, each with a clear-cut circuit region: electrical, mechanical, acoustical. The power of analogies is that any single concept which triggers the imagination (e.g. the concept "*velocity*") can be mapped from one domain to another without its perceived character changing. (We would never confuse velocity with deflection). But the units *do* change: the three velocity-like concepts *A*, *v*, *U* have different units [coulomb/s], [m/s], [m^3/s] (Table I).

Table I shows how the various expressions of any single notion are related, using the conventional symbols as far as possible. In choosing symbols for *quantities* we have some freedom (*vel*, *v*, *u* for velocity). But not in symbols for *units.* The use of S. I. units[1] (and their symbols) is mandatory (velocity is [m/s], not [miles/hour] and not [m/sec]). On the other hand symbols for *quantities* are just recommendations.

In mechanics there are 2 analogies:
(1) impedance analogy (volts ≡ force; el. current ≡ velocity),
(2) mobility analogy (volts ≡ velocity; el. current ≡ force).

We use analogy (1). It is more intuitive [6] and the extension to the acoustic domain, in which it is superior (topological similarity between circuit and physical arrangement) is easy: volts ≡ pressure, and I ≡ volume current. Nevertheless it must be admitted that (2) is topologically advantageous in mechanics. But it takes time to learn it.

We need in future to avoid confusion between two types of acoustic quantities: (1) *purely acoustic* quantities, e.g. acoustic stiffness K_{acoust} [Pa/(m^3/s)] common in general acoustics, but also sometimes used in reed literature[2]), and (2) *hybrid* quantities traditional in reed literature and appropriate here. *Impedances* (and impeding quantities such as stiffness) combine two different categories of variables[3]: "potential" and "motional" (here *pot* and *mot*). Examples of *pot* are force and pressure. Examples of *mot* are excursion, volume displacement, velocity, volume current [m^3/s].

"Pure-acoustic" stiffness K_{acoust} [Pa/(m^3/s)] is not the same as hybrid stiffness K_a [Pa/m] where the subscript "a" means "acoustic", but refers only to the *pot* quantity, the *mot* quantity in the denominator remaining mechanical. And yet both are called "acoustical". To derive the hybrid K_a from K_{mech} one multiplies K_{mech} by area *once*. To derive K_{acoust} one multiplies *twice*. In much of the reed literature the reader needs to do a lot of detective work to find out which one the author meant: pure acoustic or hybrid acoustic.

Table II collects more notions, mainly those used in the literature for acoustics and mechanics of wind instruments (see [4] and [5] and the references given there). Another source of confusion is the subscript *a*, which sometimes stands for "area" and sometimes for "acoustical".

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[1] S.I. stands for Système International d'Unités)

[2] Fletcher [7], in his admittance Y_r (with negative real part), uses pure acoustic units [(m^3/s)/Pa].

[3] If *pot* is in the numerator *mot* is in the denominator: R_a = *pot*/*mot* = pressure/velocity [Pa/(m/s)].

Electricity			Mechanics			Acoustics		
Quantity	Symbol	Unit	Quantity	Symbol	Unit	Quantity	Symbol	Unit
Voltage	*U, u*	V	Force	*F*	N	Pressure	*p, P*	Pa = N/m^2
Current	*I, i*	A = C/s	Velocity	*v, u*	m/s	Volume Flow	*u, U, q*	m$^{3/}$s
Charge	*Q, q*	C = As	Excursion	*s, x, y*	M	Vol. Displacement	*Vol*	m^3
Inductance	*L*	H = Vs/A	Mass	*m*	Kg	Inertance, a. Mass	*M*, M_a	Pa.s^2/m^3 = kg/m^4
Capacitance	*C*	F = As/V	m. Compliance	C_m	m/N	a. Compliance	*C*, C_a	m^3/Pa
1/ Capacitance	*1/C*	1/F=V/As	m. Stiffness	K_m	N/m	a. Stiffness	*K*, K_a	Pa/m^3
Resistance	*R*	Ohm = V/A	m. Resistance	R_m	N/(m/s)=Ohm_m	a. Resistance	*R*, R_a	Pa/(m^3/s) = Ohm_a
Impedance	*Z*	Ohm = V/A	m. Impedance	Z_m	N/(m/s)=Ohm_m	a. Impedance	*Z*, Z_a	Pa/(m^3/s) = Ohm_a
Admittance	*Y*	Mho = A/V	m. Admittance	Y_m	(m/s)/N=Mho_m	a. Admittance	*Y*, Y_a	(m^3/s)/Pa = Mho_a
Power	*P*	W =VA	Power	*P*	W=Nm/s	Power	*P*	W=Pa.m$^{3/}$s = N.m/s
Energy	*W, E*	J = Ws	Energy	*W, E*	J=Nm	Energy	*W, E*	J= Pa.m^3 = N.m

Table I, Electro-mechanical-acoustical analogies

Hybrid System for impedance related quantities		
Quantity	Symbol	Unit
Hybrid Mass (or Hybrid Inertance)	M_a	kg/m^2
Hybrid Compliance	C_a	m/Pa
Hybrid Stiffness	K_a	Pa/m
Hybrid Friction	R_a	Pa.s/m
Hybrid Impedance	Z_a	Pa.s/m
Hybrid Admittance	Y_a	m/Pa.s

Table II, Hybrid system

Electrical definitions in MicroCAP			Used in these simulations and/or recommended by the authors		
Quantity	Symbol	Unit *)	Quantity	Symbol (.define)	Unit *)
Voltage	V, v	V	Mechanical Force	*Force*	N
Voltage	V, v	V	Acoustic Pressure	P_m, P_b, *delta_p*	Pa = N/m^2
Current	I, i	A	Mechanical Velocity	*velocity*	m$^/$s
Current	I, i	A	Acoustic Volume Flow	*volflow*	m$^{3/}$s
Charge	Q, q	C = As	Mechanical Excursion	*yL*	m
Charge	Q, q	C = As	Acoust. Volume Displacement	*vol*	m^3
Inductance	L1, L2, ...	H = Vs/A	Mechanical Mass	L_{mech}	Kg
			Hybrid Mass	M_a	kg/m^2
Inductance	L1, L2, ...	H = Vs/A	Acoustic Mass, Inertance	M_{aa}	Pa.s^2/m^3 = kg/m^4
Capacitance	C1, C2, ...	F = As/V	Mechanic Compliance	C_{mech}	m/N
			Hybrid Compliance	C_a	m/Pa
Capacitance	C1, C2, ...	F = As/V	Acoustic Compliance	C_{aa}	m^3/Pa
1/ Capacitance	1/C1, 1/C2, ...	1/F = V/As	Mechanical Stiffness	K_{mech}	N/m
			Hybrid Stiffness	K_a	Pa/m
1/ Capacitance	1/C1, 1/C2, ...	1/F = V/As	Acoustic Stiffness	K_{aa}	Pa/m^3
Resistance	R1, R2, R3, ...	Ohm = V/A	Mechanical Friction	R_{mech}	kg/s = Ohm_m
			Hybrid Friction	R_a	Pa.s/m
Resistance	R1, R2, R3, ...	Ohm = V/A	Acoustic Resistance	R_{aa}	Pa.s/m^3 = Ohm_a
Impedance	Zel **)	Ohm = V/A	Mechanical Impedance	Z_{mech}	kg/s = Ohm_m
			Hybrid Impedance	Z_a	Pa.s/m
Impedance	Zel **)	Ohm = V/A	Acoustic Impedance	Z_{aa}	Pa/(m^3/s) = Ohm_a
			Damping factor, R_a/M_a (=R_{aa}/Maa =R_{mech}/L_{mech})	*g or* g_{mech}	1/s
			Reed damping factor =1/Q	κ_r used by Fletcher [7]	dimensionless

*) Not displayed in MicroCAP, **) Not defined in MicroCAP

Table III, Examples of symbols recommended by the authors to avoid ambiguity

Often in computer programming other (mostly longer) symbols are used, to avoid short variables that (1) might be reserved by the programming language for other purposes, and (2) would make the search function tedious.

Table III gives an overview of the definitions used as well as recommendations by the authors. The subscript *a* used *once* (as in K_a), means a hybrid impeding quantity. Used *twice* it means a purely acoustic impeding variable. This code reflects how often K_{mech} must be multiplied by *A* (reed area):
$K_a = K_{mech} \times A$,
$K_{aa} = K_{mech} \times A \times A$.

Note that explicit mention of the acoustic system is not always needed. For example the damping factor "*g*" ($=R_a/M_a$) is always the same whether *mech*, *a* or *aa*. The same applies to quality factor and resonance frequency.

In the models used here there is an ideal transformer that links the acoustical and the mechanical domains. Unlike the conventional transformer whose turns ratio $1{:}N$ is dimensionless, in this one *N* is numerically equal to the area acting as interface. The side where 1 occurs is the acoustic part, N occurs on the mechanical side. This technique leads to a clear distinction of acoustics and mechanics. Another difference is that, whereas a conventional transformer can only transform AC signals, an ideal transformer can transform DC just as well.

Remark: There are four different area parameters:

(1) The mechanical area is that part of the reed that is free to oscillate and in the lumped model constitutes the so-called effective area, which is only meaningful when compared with a distributed reed model with approximately the same properties.

(2) The inner acoustic area lies inside the mouthpiece and is constant.

(3) The outer acoustic area is usually smaller than the inner one, as the lower lip of the player hinders the air pressure to act onto the rear part of the reed.

(4) The fourth area is not a solid surface. It is the cross-sectional area of the slit aperture perpendicular to air flow. This slit between mouthpiece tip and reed tip plays a role in introducing gaseous viscosity and inertial effects (inertance of the air mass). In our models this slit is always placed on the acoustical side of the circuit, but was omitted here for simplicity.

6. REFERENCES

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